LECTURE NOTES

LAND SURVEY -2

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6th SEMESTER

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TACHEOMETRIC SURVEYING Dt-23.04.21

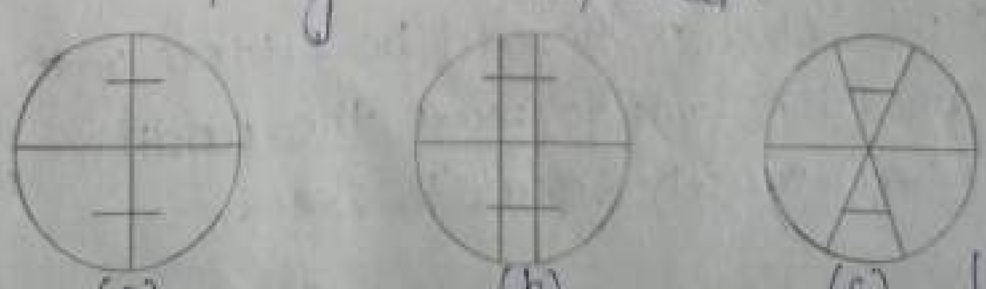
INTRODUCTION

Tacheometry is a branch of surveying in which horizontal and vertical distances are determined by taking angular observations with an instrument known as tacheometer. The chaining operation is completely eliminated in such survey. Tacheometric surveying is adopted in rough and difficult terrain where direct levelling and chaining are either not possible or very tedious. It is also used in location survey for railways, roads, reservoirs, etc. Though not very accurate, tacheometric surveying is very rapid and a reasonable contour map can be prepared for investigation works within a short time on the basis of such survey.

(1) Instruments used in tacheometry :-

(a) The Tacheometer :-

It is nothing but a transit theodolite fitted with a stadia diaphragm and an anallatic lens. Figure 11.1 shows the different forms of stadia diaphragm commonly used :



(a) (b) (c) (11.1)

(b) The levelling staff and stadia rod :-

For short distances, ordinary levelling staves are used. The levelling staff is normally 4m long, and can be folded into three parts. The graduations are so marked that a minimum reading of 0.005 or 0.001 m, can be taken.

for long sights a specially designed graduated rod is used, which is known as a stadia rod. It is also 4m long, and may be folded or telescopic. The graduations are comparatively bold and clear and the minimum reading that can be taken as 0.001 m.

(2) characteristics of tacheometer :-

(a) The value of the multiplying constant $\frac{f}{i}$ is should be 100.

(b) The telescope should be powerful, having a magnification of 20 to 30 diameters.

(c) The aperture of the objective should be of a 35 to 45 mm diameter for there to be a bright image.

(d) The telescope should be fitted with an anallatic lens to make the additive constant $(f+d)$ exactly equal to zero.

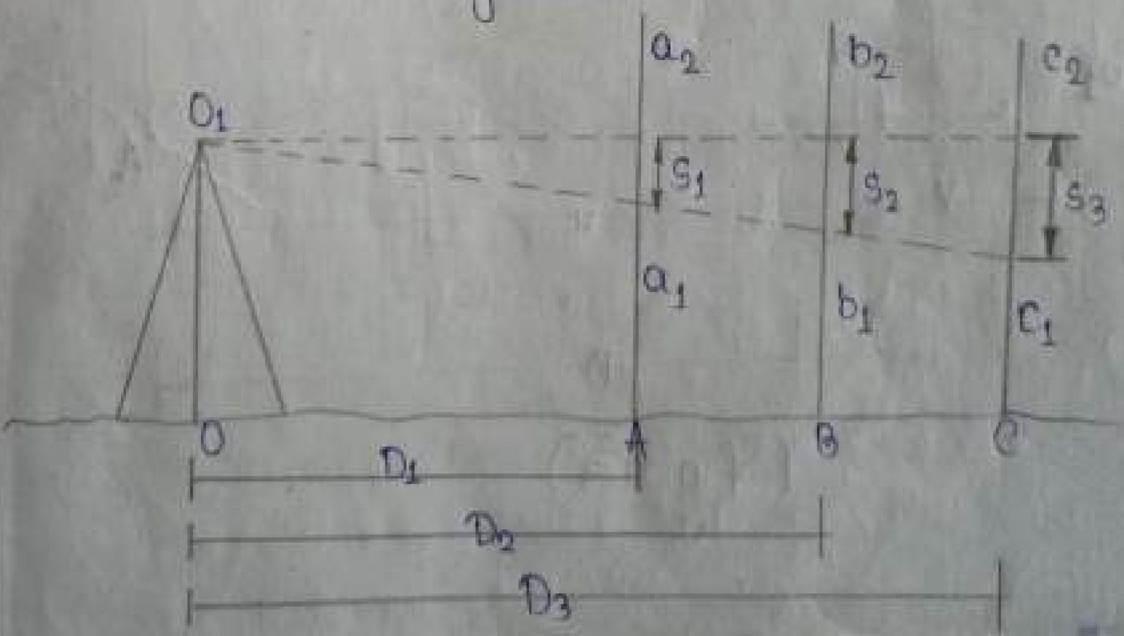
(2) the eye-piece should be of greater magnifying power than usual, so that it is possible to obtain a clear staff reading from a long distance.

(3) principle of Tacheometry :-

The principle of tacheometry is based on the property of isosceles triangles, where the ratio of distance of the base from the apex and the length of the base is always constant.

In fig. 11.2 $qO_1a_1a_2$, $qO_1b_1b_2$, and $qO_1c_1c_2$ are all isosceles triangles where D_1, D_2 and D_3 are the distances of the bases from the apices and s_1, s_2 and s_3 are the lengths of the bases (staff intercepts).

fig 11.2



so; according to the stated principle,

$$\frac{D_1}{S_1} = \frac{D_2}{S_2} = \frac{D_3}{S_3} = \frac{f}{i} \text{ (constant)}$$

The constant $\frac{f}{i}$ is known as the multiplying constant.

Where,

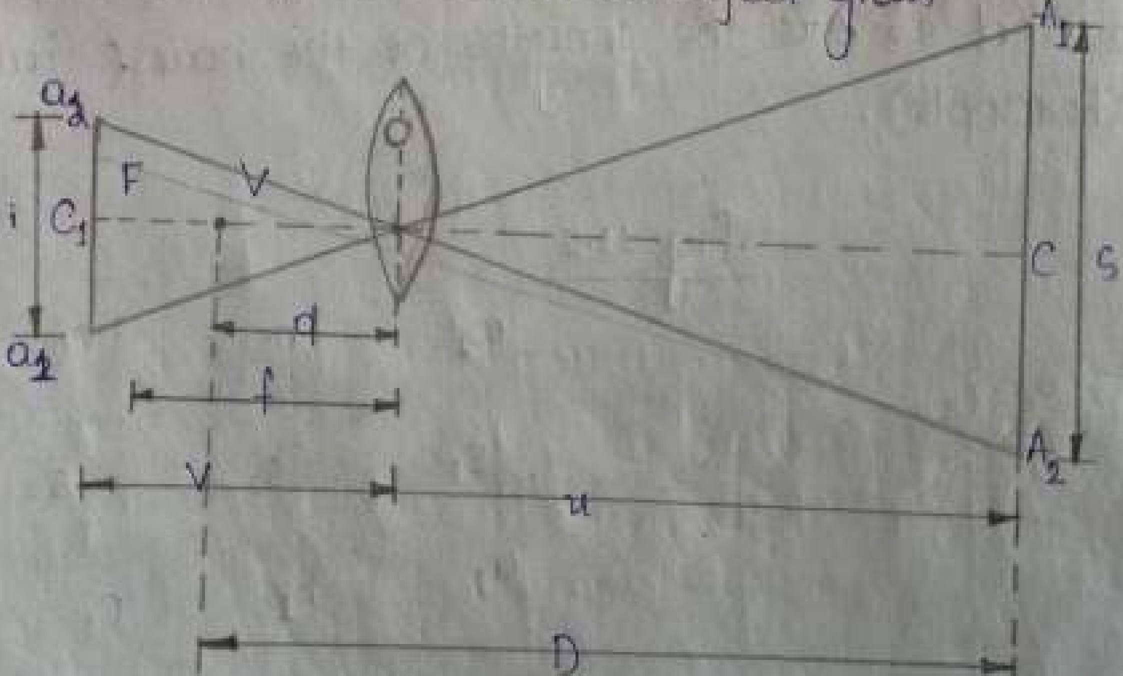
f = focal length of objective

i = stadia intercept

THEORY OF STADIA TACHEOMETRY

The following is the notation used in stadia tacheometry. (fig-11.3)

O = optical centre of object glass



(fig 11.3)

A_1, A_2, C = readings on staff cut by three hairs
a_1, a_2, c = bottom, tops and central hairs of diaphragm.

$a_1 a_2 = i$ = length of image

$A_1 A_2 = S$ = staff intercept

F = Focus

V = vertical axis of instrument

f = focal length of object glass

d = distance between optical square centre and vertical axis of the instrument.

u = distance between optical centre and staff

v = distance between optical centre and image

from similar triangles $a_1 O a_2$ and $A_1 O A_2$;

$$\frac{i}{s} = \frac{v}{u} \text{ or } v = \frac{iu}{s}$$

from the properties of lenses,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Putting the value of v in Eq. (2)

$$\frac{1}{iu/s} + \frac{1}{u} = \frac{1}{f}$$

(OR)

$$\frac{s}{iu} + \frac{1}{u} = \frac{1}{f}$$

OR

$$\frac{1}{u}\left(\frac{s}{i} + 1\right) = \frac{1}{f}$$

OR

$$u = \left(\frac{s}{i} + 1\right) f$$

But,

$$D = u + d$$

So,

$$D = \left(\frac{s}{i} + 1\right) f + d$$

$$= \frac{s}{i} \times f + f + d$$

$$= \left(\frac{f}{i}\right) \times s + (f + d)$$

The quantities (f/i) and $(f+d)$ are known as tacheometric constants. $\left(\frac{f}{i}\right)$ is called the multiplying constant, as already stated, and $(f+d)$ the additive constant. By adopting an anallatic lens in the telescope of a tacheometer the multiplying constant is made 100, and the additive constant zero. However, in some of tacheometers the additive constants are not exactly zero, but vary from 30 cm to 60 cm (which are generally mentioned in the catalog supplied by the manufacturer.

DETERMINATION OF TACHEOMETRIC OR STADIA CONSTANT

The constants may be determined by

(1) Laboratory measurement
(2) Field measurement

(1) Laboratory Measurement

The focal length f of the lens can be determined by means of an optical bench, according to the equation:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

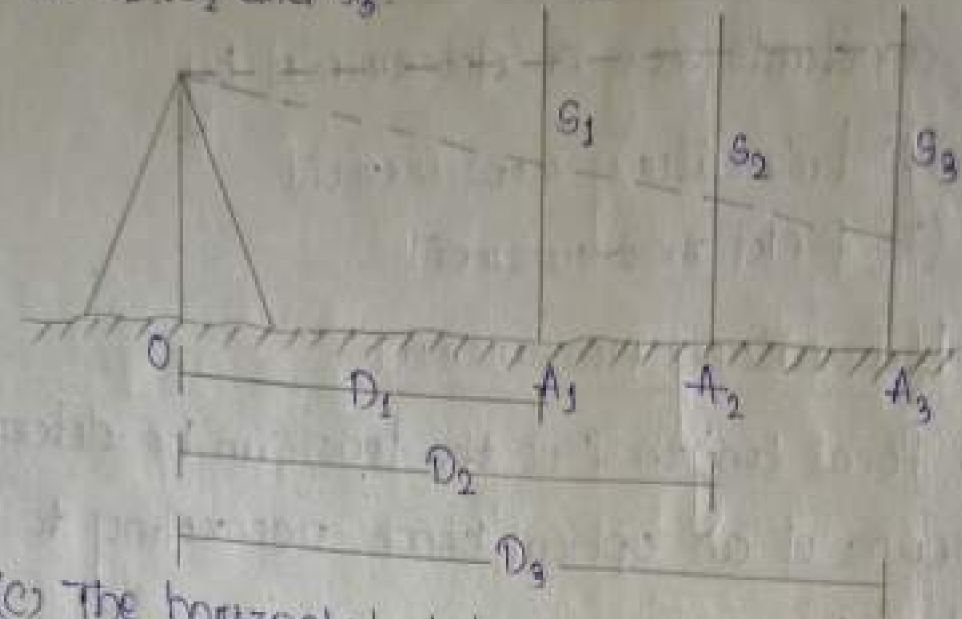
The stadia intercept i can be measured from the diaphragm with the help of a Vernier Caliper. The distance d between the optical centre and the vertical axis of the measurement can also be determined / measured.

In this manner, the multiplying $\left(\frac{f}{i}\right)$ and additive $(f+d)$ constants can be calculated.

(2) Field Measurement

(a) A fairly level ground is selected. The tacheometer is set up at O and pegs are fixed at A_1, A_2 and A_3 known distances apart. (11.4)

(b) The staff intercepts (stadia hair readings) are noted at each of the pegs. Let these intercepts be s_1, s_2 and s_3.



(c) The horizontal distances of the pegs from O are accurately measured. Let these distances be D_1, D_2 and D_3.

(d) By substituting the values of $D_1, D_2 \ldots$ and $s_1, s_2 \ldots$ in the general equation

$$D = \left(\frac{f}{i}\right) s + (f+d)$$

We get a number of equations, as follows:

$$D_1 = \left(\frac{f}{i}\right) s_1 + (f+d)$$

$$D_2 = \left(\frac{f}{i}\right) s_2 + (f+d) \text{ and so on.}$$

(e) By solving the equations in pair, several values of $\left(\frac{f}{i}\right)$ and $(f+d)$ are obtained. The mean of these values gives the required constant.

METHODS OF TACHEOMETRY

Tacheometry involves mainly two methods:

1. The stadia method
2. The tangential method

1. The stadia method

In this method the diaphragm of the tacheometer is provided with two stadia hairs (upper and lower). Looking through the telescope the stadia hair readings are taken. The difference in these readings gives the staff intercept. To determine the distance between the station and the staff, the staff intercept is multiplied by the stadia constant (i.e. multiplying constant, 100). The stadia method may, in turn, be of two kinds.

(a) The fixed Hair Method-

The distance between the stadia hairs is fixed in this method, which is the one commonly used. When the staff is sighted through the telescope, a certain portion of the staff is intercepted by the upper and lower stadia. The value of the staff intercept varies with the distance. However, the distance between the station and the staff can be obtained by multiplying the staff intercept by the stadia constant.

(b) The movable Hair Method :

The stadia hairs are not fixed in this method. They can be moved or adjusted by micrometer screws. The staff is provided with two targets, or vanes a known distance apart. During observation, the distances between stadia hairs is so adjusted that the upper hair bisects the upper target and the lower hair bisects the lower target. The variable stadia intercept is measured and the required distance is then computed.

This method is not generally used.

2. The Tangential Method :

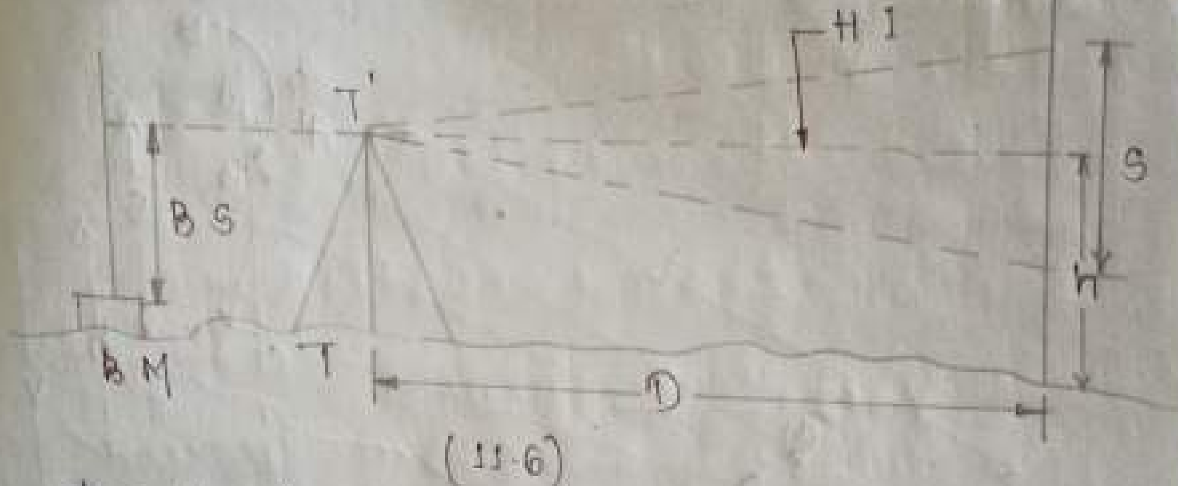
In this method, the diaphragm of the tacheometer is not provided with stadia hair. The readings are taken by the single horizontal hair.

The staff consists of two vanes or targets a known distance apart. To measure the staff intercepts, two pointings are required. The angles of elevation or depression are measured and their tangents are used for finding the horizontal distances and elevation.

This method is also not generally used. The stadia methods requires only one observation, but the tangential method requires two pointings of the telescope.

FIXED HAIR METHOD

Case 1: When line of sight is horizontal and the staff is held vertically.



(11.6)

When the line of sight is horizontal, the general tacheometric equation for distance is given by

$$D = \left(\frac{f}{i}\right) s + (f+d)$$

The multiplying Constant $\left(\frac{f}{i}\right)$ is 100, and additive Constant $(f+d)$ is generally zero.

RL of staff station $P = HI - h$

Where, $HI = RL$ of $BM + BS$

h = central hair reading

(HI = height of instrument)

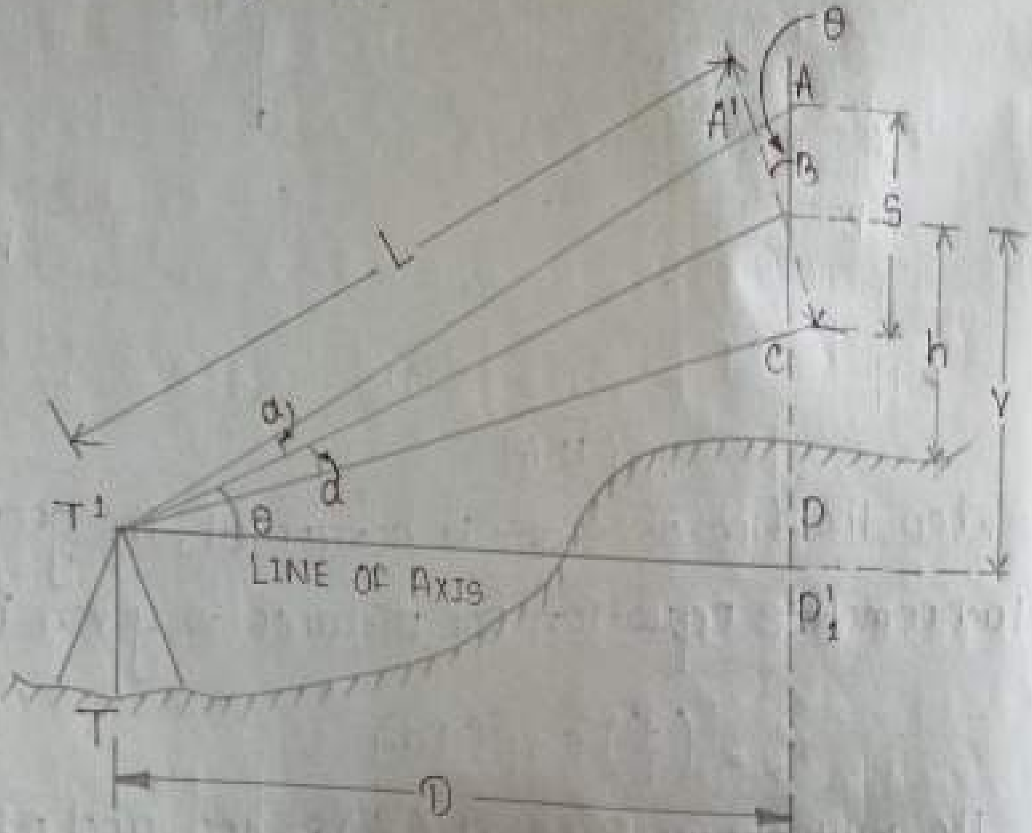
(BS = backsight)

Case-II

When line of sight is inclined, but staff is held vertically.

- Here, the measured angle may be the angle of elevation or that of depression.

(a) Considering Angle of Elevation (positive)



(fig 11.7)

T = instrument station

T_1 = axis of instrument

P = staff station

A, C, B = position of staff cut by hairs

s = AC = staff intercept

h = central hair readings

v = vertical distance between instrument axis and central hair

D = horizontal distance between instrument and staff

L = inclined distance between instrument axis and B

θ = angle of elevation

α = angle made by outer and inner rays with central ray.

A'C' is drawn perpendicular to the central ray, T_1B.

Now, inclined distance,

$$L = \frac{f}{i}(A'C') + (f+d)$$

Horizontal distance $D = L\cos\theta$

$$= \frac{f}{i}(A'C')\cos\theta + (f+d)\cos\theta$$

Now A'C' is to be expressed in terms of AC (i.e. s).

In Δs ABA' and CBC',

$$\angle ABA' = \angle CBC' = \theta$$

$$\angle AA'B = 90° + \alpha$$

$$\angle BC'C = 90 - \alpha$$

The angle α is very small.

$\angle AA'B$ and $\angle BC'C$ may be taken equal to $90°$

so,

$$AC' = AC \cos\theta = s\cos\theta$$

from Eq.(1),

$$D = \frac{f}{i}(s\cos\theta)\cos\theta + (f+d)\cos\theta$$

$$D = \frac{f}{i} \times s\cos^2\theta + (f+d)\cos\theta \qquad (1)$$

Again,

$$V = L\sin\theta$$

$$= \left\{\frac{f}{i} \times s\cos\theta + (f+d)\right\}\sin\theta$$

$$= \frac{f}{i} \times s\cos\theta\sin\theta + (f+d)\sin\theta$$

$$V = \frac{f}{i} \times \frac{s \times \sin 2\theta}{2} + (f+d)\sin\theta \qquad (2)$$

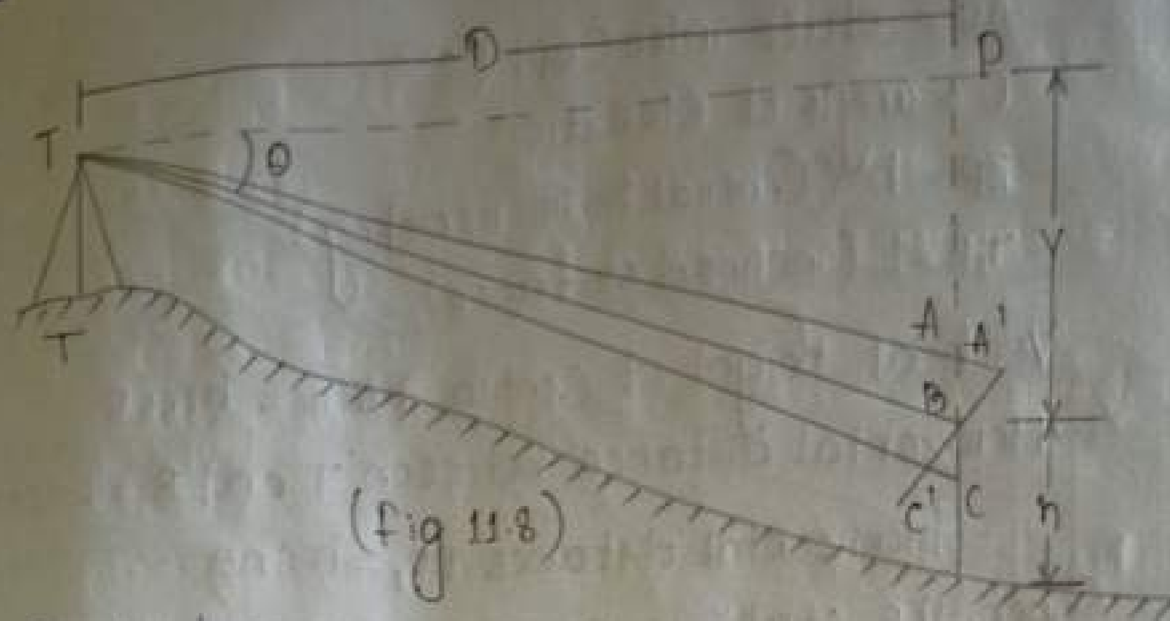
$$V = D\tan\theta$$

RL of staff station P = RL of axis of instrument $+ V - h$ (3)

(b) Considering angle of depression (negative) In this case also (fig 11.8), the expressions for D and V are same as in (a). That is,

$$D = \frac{f}{i} \times s\cos^2\theta + (f+d)\cos\theta \qquad (4)$$

$$V = \frac{f}{i} \times \frac{s \times \sin^2\theta}{2} + (f+d)\sin\theta \qquad (5)$$



(fig 11.8)

RL of staff station, P = RL of axis of instrument
−V−h (6)

Case-III

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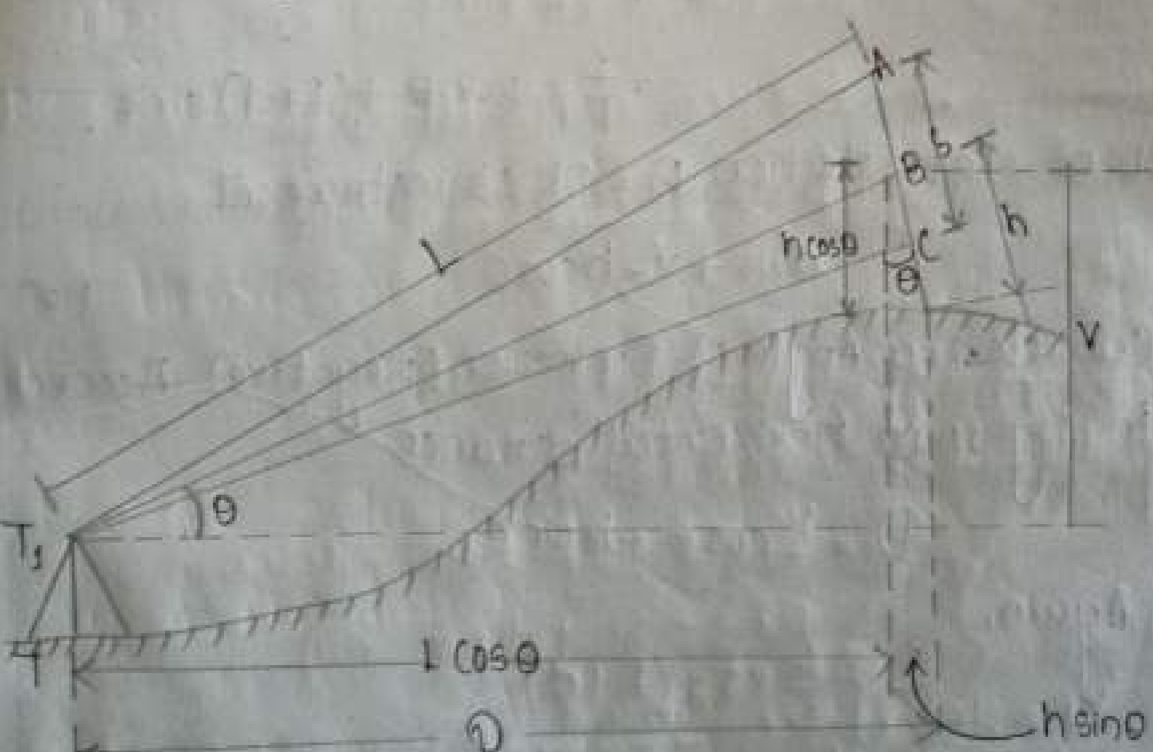
Line of sight inclined, but staff normal to it

(a) Considering Angle of Elevation (positive)



(Fig 11.9)

AC = staff intercept (s)
θ = angle of elevation
BP = h (Central hair reading)
T_1B = L (inclined distance)

Vertical height of Central hair = $h\cos\theta$

Horizontal distance between T and B = $L\cos\theta$

Horizontal distance $PP_1 = h\sin\theta$

Since the staff is perpendicular to the line of Collimation,

$$L = \frac{f}{i} \times s + (f+d)$$

Horizontal distance $D = L\cos\theta + h\sin\theta$

$$= \frac{f}{i} \times s\cos\theta + (f+d)\cos\theta + h\sin\theta$$

Vertical distance $V = L\sin\theta$

$$= \frac{f}{i} \times s\sin\theta + (f+d)\sin\theta \quad (8)$$

RL of staff station P = RL of instrument axis + V − $h\cos\theta$ (9)

(b) Considering Angle of Depression (Negative) According to fig 11.10, horizontal distance

$$D = L\cos\theta - h\sin\theta$$

Again,

$$L = \frac{f}{i} \times s + (f+d)$$

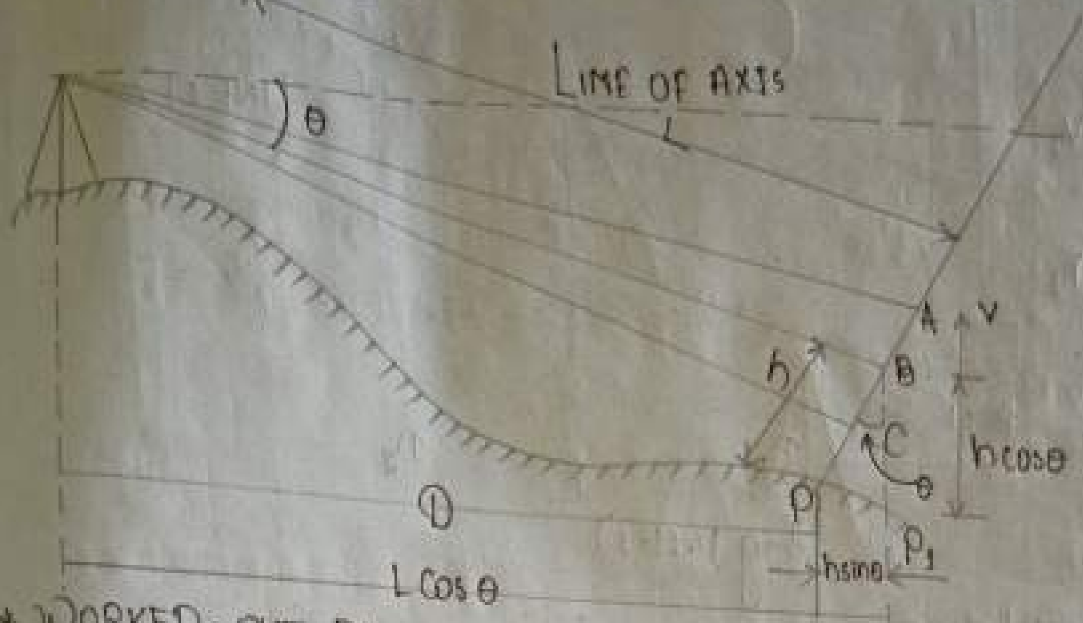
$$\therefore D = \frac{f}{i} \times s \times \cos\theta + (f+d)\cos\theta - h\sin\theta \quad (10)$$

vertical distance,

$$V = L \sin\theta$$

$$\therefore V = \frac{f}{i} \times s \sin\theta + (f+d)\sin\theta \qquad (11)$$

RL of P = RL of instrument axis − V − h cosθ



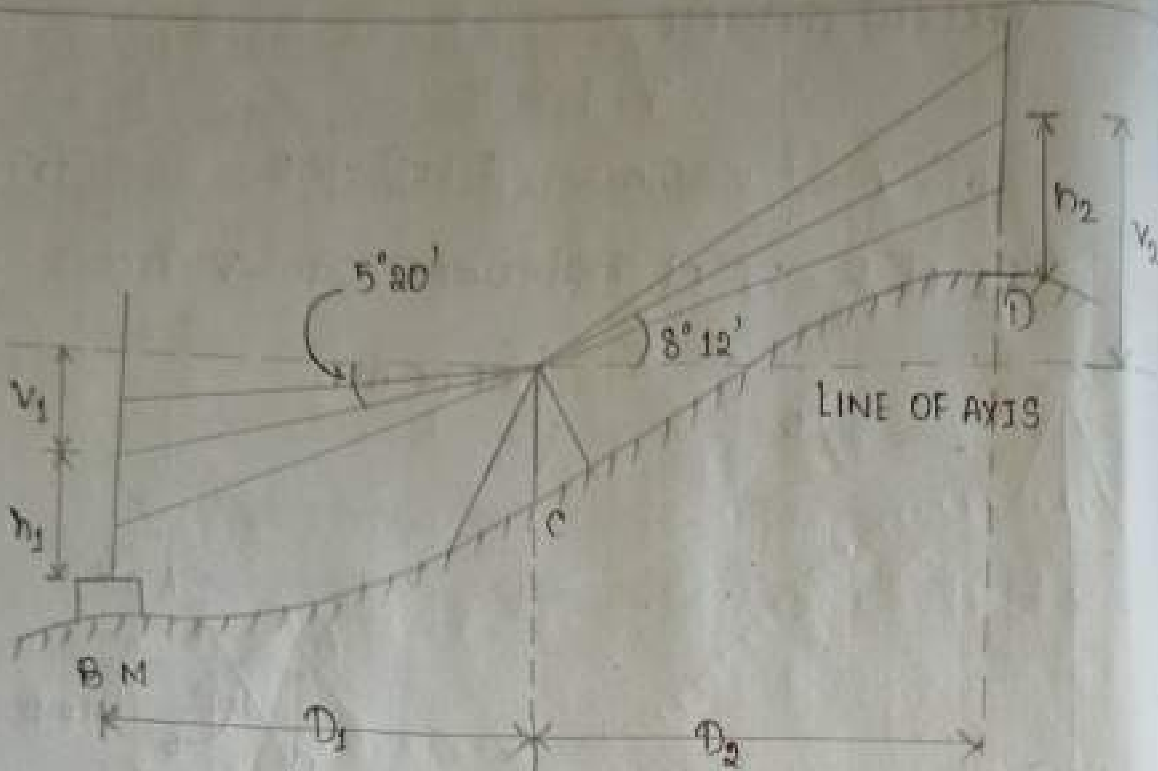
* WORKED-OUT PROBLEMS ON FIXED HAIR METHOD OF TACHEOMETRY :-

Problem 1

A tacheometer was set up at a station C and the following readings were obtained on a staff vertically held.

Inst. station	Staff Station	Vertical angle	Hair readings (m)	Remark
C	BM	−5° 20′	1.750, 1.800, 2.450	RL of BM
C	D	+8° 12′	1.750, 1.500, 2.250	= 750.50m

Calculate the horizontal distance CD and RL of D, when the constants of instrument are 100 and 0.15.



(fig D.11.1)

Solution

When the staff is held vertically, the horizontal and vertical distances are given by relations

$$D = \frac{f}{i} \times S\cos^2\theta + (f+d)\cos\theta$$

$$V = \frac{f}{i} \times S \times \frac{\sin 2\theta}{2} + (f+d)\sin\theta$$

Here,

$$\frac{f}{i} = 100 \text{ and } (f+d) = 0.15$$

In the first observations,

$$S_1 = 2.450 - 1.150 = 1.300\,m$$

$$\theta_1 = 5^\circ 20' \text{ (depression)}$$

$$V_1 = 100 \times 1.300 \times \frac{\sin 10^\circ 4'}{2} + 0.15 \times \sin 5^\circ 20' = 12.045\,m$$

In the Second observation,

$$S_2 = 2.250 - 0.750$$
$$= 1.500$$
$$\theta_2 = 8.12' \text{ (elevation)}$$

$V_2 = 100 \times 1.500 \times \frac{\sin 16^\circ 24'}{2} + 0.15 \times \sin 8^\circ 12' = 21.197 \text{ m}$

$D_2 = 100 \times 1.500 \times \cos^2 8^\circ 12' + 0.15 \times \cos 8^\circ 12'$

$= 147.097 \text{ m}$

RL of instrument axis = RL of BM + h_1 + V_1

$= 750.500 + 1.800 + 12.045$

$= 764.345 \text{ m}$

RL of D = RL of inst. axis + $V_2 - h_2$

$= 764.345 + 21.197 - 1.500$

$= 784.042 \text{ m}$

So, the distance OD = 147.097 m and RL of D = 784.042 m

Problem-2

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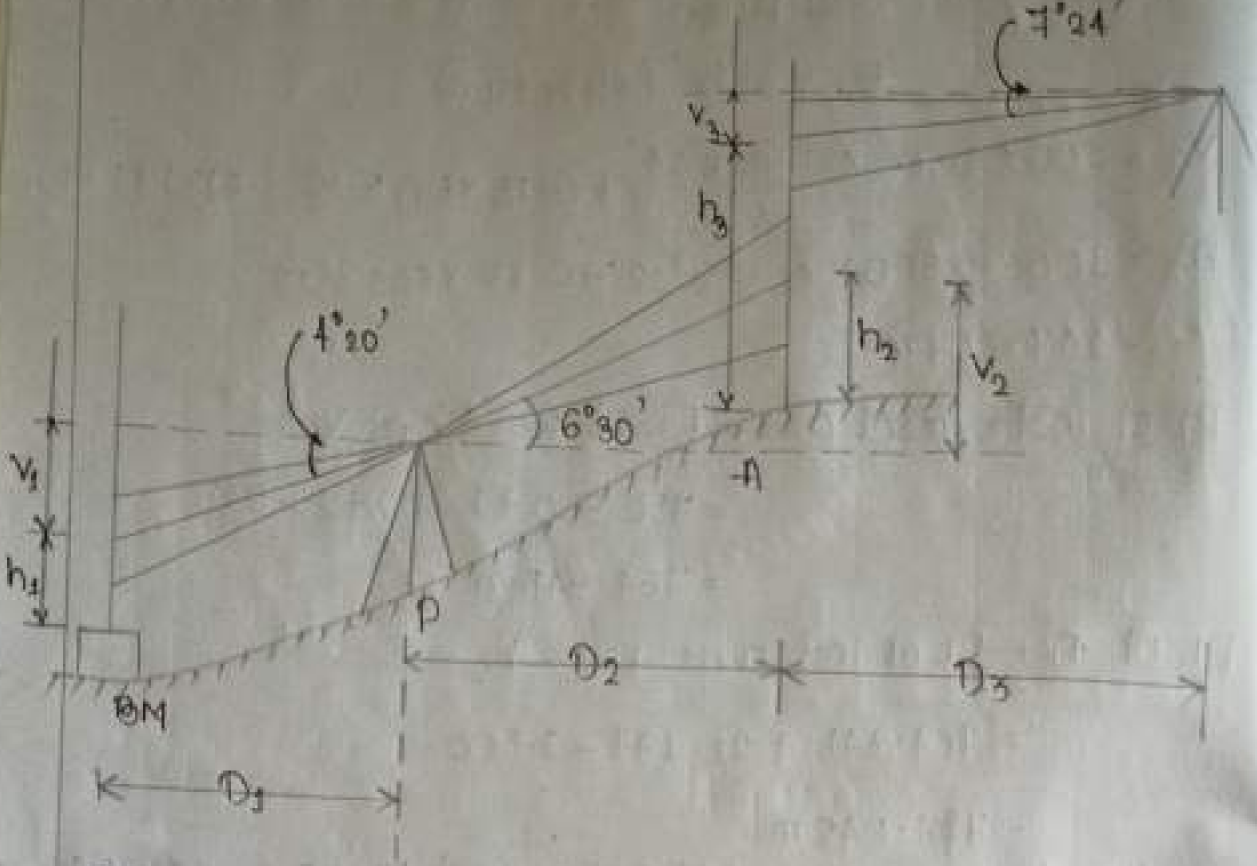
The following observations were taken with a tacheometer fitted with an anallatic lens, the staff being held vertically. The constant of the tacheometer is 100.

Inst. station	Height of instrument	staff station	Vertical angle	Staff reading (m)	Remark
P	1.255	BM	$-4^\circ 20'$	1.325, 1.825, 2.325	RL of BM = 255.750 m
P	1.255	A	$+6^\circ 30'$	0.850, 1.600, 2.350	
B	1.450	A	$-7^\circ 24'$	1.715, 2.315, 2.915	

Calculate the RL of B and the distance between A and B.



Solution :-

Here,

Multiplying Constant, $\frac{f}{i} = 100$ and Additive Constant $f + d = 0$

Since,

the staff is held vertically, the vertical distance is given by

$$V = \frac{f}{i} \times S \times \frac{\sin 2\theta}{2}$$

In the observations,

$$V_1 = 100(2.335 - 1.325) \times \frac{\sin 8°40'}{2}$$

$$= 7.534 \text{ m}$$

In the second observation,

$$V_2 = 100(2.350 - 0.850) \times \frac{\sin 13^\circ 0'}{2}$$

$$= 16.871\ m$$

In the third observation,

$$V_3 = 100(2.915 - 1.715) \times \frac{\sin 14^\circ 18'}{2}$$

$$= 15.826\ m$$

RL of axis when Inst. at P = RL of BM + h_1 + V_1

$= 255.750 + 1.825 + 7.534$

$= 265.109\ m$

RL of A $= 265.109 + V_2 - h_2$

$= 265.109 + 16.871 - 600$

$= 280.380\ m$

RL of axis when inst. at B $= 280.380 + h_3 + V_3$

$= 280.380 + 2.315 + 15.326$

$= 298.021\ m$

RL of B $= 298.021 - h_4$

$= 298.021 - 1.450$

$= 296.571\ m$

Distance between A and B, $D_3 = 100(2.915 - 1.715) \times \cos^2 7°24$

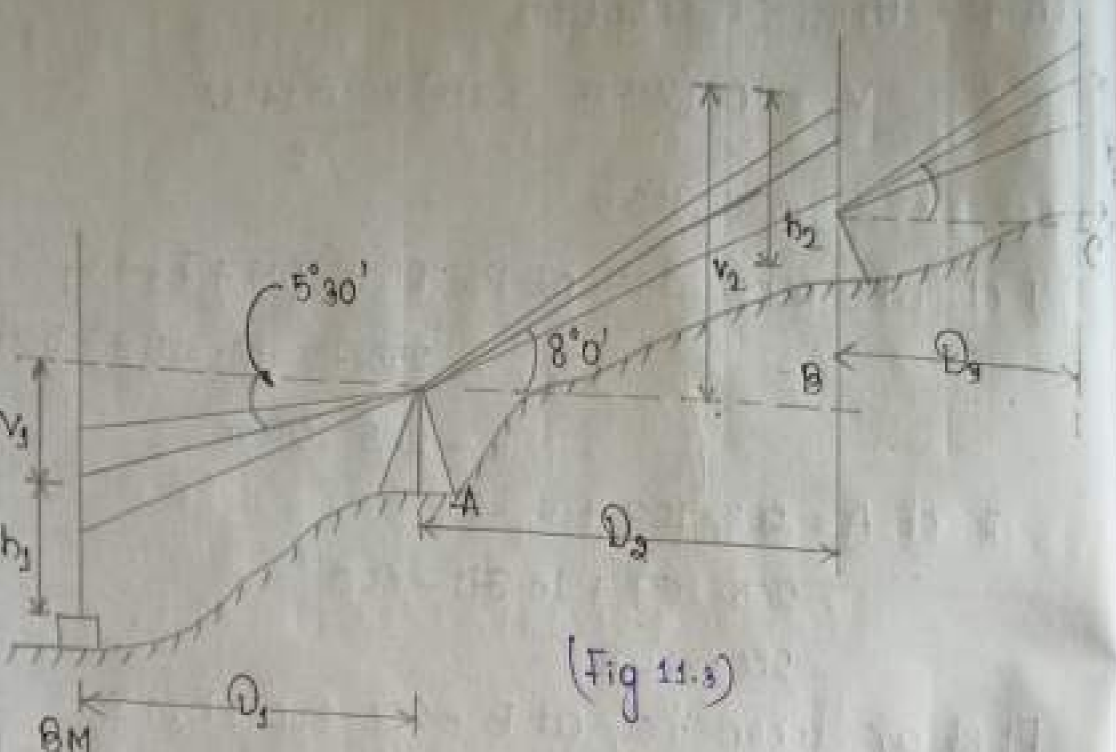
$= 118.009\ m$

Problem 3 :-

The following observation were made in a tacheometric survey.

Inst station	Height of axis	Staff station	Vertical angle	Hair readings(m)	Remark
A	1.845	BM	$-5^\circ 30'$	0.905, 1.455, 2.005	RL of BM
A	1.845	B	$+6^\circ 0'$	0.755, 1.655, 2.555	=450.500m
B	1.550	C	$+10^\circ 0'$	1.500, 2.250, 3.000	

Calculate the RLs of A, B and C and the horizontal distances AB and BC. The tacheometer is fitted with an anallatic lens and the multiplying constant is 100.



(Fig 11.3)

<u>Solution</u>

Here,

$$\frac{f}{i} = 100 \quad \text{and} \quad (f+d) = 0$$

Since the staff is held vertically,

$$\text{Horizontal distance } D = \left(\frac{f}{i}\right) \times S\cos^2\theta$$

$$\text{Vertical distance } V = \left(\frac{f}{i}\right) \times S \times \frac{\sin 2\theta}{2}$$

In the first observation,

$$V_1 = 100 \times (2.005 - 0.905) \times \frac{\sin 11^\circ}{2}$$

$$= 10.494 \text{ m}$$

$$D_1 = 100 \times (2.005 - 0.905) \times \cos^2 5^\circ 30'$$

$= 108.989\ m$

In the second observation,

$$V_2 = 100(2.555 - 0.755) \times \frac{\sin 16^\circ}{2}$$

$$= 24.807\ m$$

$$D_2 = 100(2.555 - 0.755) \times \cos^2 8^\circ$$

$$= 176.514\ m$$

In the third observation,

$$V_3 = 100 \times (3.000 - 1.500) \times \frac{\sin 20^\circ}{2}$$

$$= 25.652\ m$$

$$D_3 = 100 \times (3.000 - 1.500) \times \cos^2 10^\circ$$

$$= 145.477\ m$$

Distance AB $= D_2 = 176.514\ m$

Distance BC $= D_3 = 145.477\ m$

RL of axis when inst. at A = RL of BM + $V_1 + h_1$

$= 450.500 + 10.494 + 1.455$

$= 462.449\ m$

RL of A $= 462.449$ height of axis

$= 462.449 - 1.345$

$= 461.104\ m$

RL of B $= 462.449 + V_2 - h_2$

$= 462.449 + 24.807 - 1.655$

$= 485.601\ m$

RL of axis when inst. at B $= 485.601 + 1.550$

$= 487.151\ m$

RL of C $= 487.151 + V_3 - h_3$

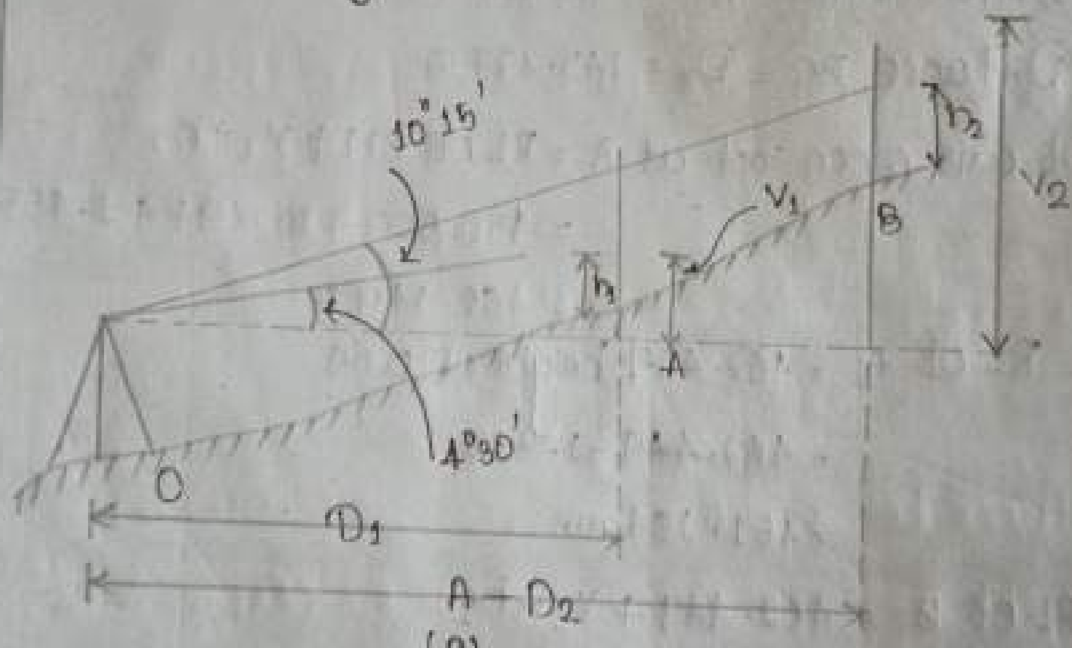
$= 487.151 + 25.652 - 2.250 = 510.553\ m$

Problem-1

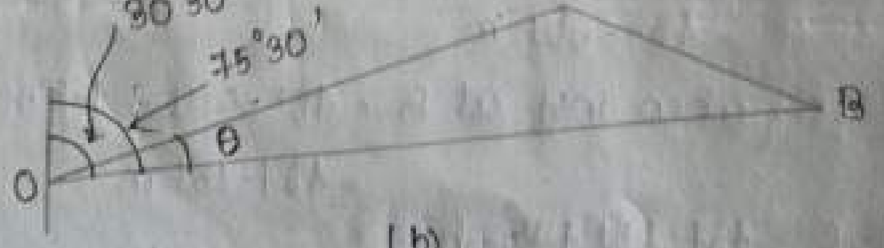
The following observation were made using a tacheometer fitted with an anallatic lens, the multiplying constant being 100.

Inst station	Height of station	Staff station	WCB	Vertical angle	Hair readings	Remarks
O	1.550	A	30°30′	4°30′	1.155, 1.755, 2.355	RL of O = 150.000
		B	75°30′	10°15′	1.250, 2.000, 2.750	

Calculate the distance AB and the RLs of A and B find also the gradient of the line AB.



(a)



(b)

In the first Observation :-

$$V_1 = 100 \times (2.355 - 1.155) \times \frac{\sin 9^\circ}{2} = 9.386 \text{ m}$$

$$D_1 = 100 \times (2.355 - 1.155) \times \cos^2 4^\circ 30' = 119.261 \text{ m}$$

In the Second observation :-

$$V_2 = 100(2.750 - 1.250) \times \frac{\sin 20^\circ 30'}{2} = 26.265 \text{ m}$$

$$D_2 = 100(2.750 - 1.250) \times \cos^2 10^\circ 15' = 145.250 \text{ m}$$

RL of axis = RL of O + height of inst.

= 150.000 + 1.550

= 151.550 m

RL of A = $151.550 + V_1 - h_1$

= 151.550 + 9.386 − 1.755

= 159.181 m.

RL of B = $151.550 + V_2 - h_2$

= 151.550 + 26.265 − 2.000

= 175.815 m.

OA = D_1 = 119.261 m.

OB = D_2 = 145.250 m.

$\theta = 75^\circ 30' - 30^\circ 30' = 45^\circ 0'$

$$AB = \sqrt{OA^2 + OB^2 - 2 \times OA \times OB \times \cos 45^\circ}$$

$$= \sqrt{(119.261)^2 + (145.250)^2 - 2 \times 119.261 \times 145.250 \times 0.707}$$

= 104.05 m.

Distance of level between

A and B = 175.815 − 159.181

= 16.634 m. (Rise from A to B)

Gradient of AB (rising) = $\frac{16.634}{104.05} = \frac{1}{6.25}$ i.e. 1 in 6.25

Problem 5 :-

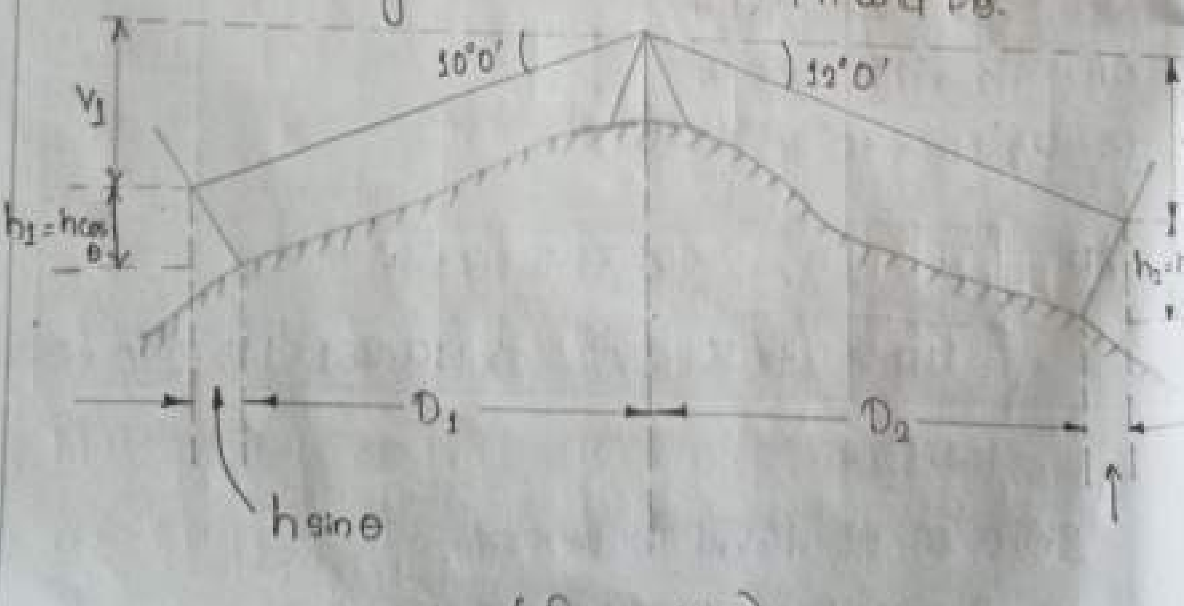
Two points A and B are on opposite sides of a summit. The tacheometer was set up at P on top of the summit and the following readings were taken.

Inst. station	Height of inst	Staff station	Vertical angle	Hair readings	Remark
P	1.500	A	$-10°0'$	1.150, 2.050, 2.950	RL of P = 450.500 m
P	1.500	B	$-12°0'$	0.855, 1.605, 2.355	

The tacheometer is fitted with an anallatic lens the multiplying constant being 100. The staff was held normal to the line of sight.

Find: (a) The distance between A and B, and

(b) The gradients of lines PA and PB.



(fig P. 11.5)

Solution

We know that when the staff is held normal to the line of sight, the vertical distance is given by

$$V = \frac{f}{i} \times S \sin\theta + (f+d) \sin\theta$$

Here, $\frac{f}{i} = 100$ and $(f+d) = 0$

$\theta_1 = 10°$ and $\theta_2 = 12°$

from eq (11),

$V_1 = \frac{f}{i} \times S \sin\theta_1 = 100 \times (2.950 - 1.150) \times \sin 10°$

$= 31.256\ m$

Similarly,

$V_2 = 100(2.355 - 0.855) \sin 12°$

$= 31.186\ m$

$h_1 = 2.050 \times \cos 10°$

$= 2.018\ m$

$h_2 = 1.605 \times \cos 12°$

$= 1.569\ m.$

RL of A inst. axis $= 450.500 + 1.500$

$= 452.000\ m$

RL of A $=$ RL of inst axis $- V_1 - h_1$

$= 452.000 - 31.256 - 2.018$

$= 418.726\ m$

RL of B $= 452.000 - V_2 - h_2$

$= 452.000 - 31.186 - 1.569$

$= 419.245\ m$

The horizontal distances are given by equation,

$$D = \frac{f}{i} \times S\cos\theta + (f+d)\cos\theta + h\sin\theta$$

Hence, $D_1 = 100 \times (2.950 - 1.150)\cos 10^\circ - 2.050 \sin 10^\circ$

$= 177.265 - 0.355$

$= 176.91$ m

$D_2 = 100(2.355 - 0.855)\cos 12^\circ - 1.605 \sin 12^\circ$

$= 146.722 - 0.333$

$= 146.389$ m

Distance between A and B $= D_1 + D_2$

$= 176.910 + 146.389$

$= 323.299$ m

Gradient of PA $= \dfrac{450.500 - 418.726}{176.910} = \dfrac{1}{5.56}$ (1 in 5

Gradient of PB (falling) $= \dfrac{450.500 - 419.245}{146.389} = \dfrac{1}{4.68}$ (1 in

Problem 8 :-

The following are the records of a tacheometric survey :

Inst. station	Staff station	Bearing	Vertical angle	Hair readings
A	B	N 30°30′ E	+ 10°0′	1.250, 1.750, 2.250
B	C	S 40°0′ E	+ 5°0′	0.950, 1.750, 2.550
C	D	S 45°0′ W	+ 8°0′	1.550, 2.150, 2.750

Multiplying constant = 100, and additive Constant = 0. The staff is held vertically. Calculate the length and bearing of DA.

Solution:

The distances are Calculated from the formula

$$D = \frac{f}{i} \times S \cos^2\theta$$

$AB = 100(2.250 - 1.250) \times \cos^2 10°$

$= 96.98\,m$

$BC = 100(2.500 - 0.950) \times \cos^2 5°$

$= 153.78\,m$

$CD = 100(2.750 - 1.550) \times \cos^2 8°$

$= 117.67\,m$

Let,

Length of DA = L and Bearing of DA = θ

Latitude

$AB = +96.98 \cos 30°30'$

$= +83.40$ (northing)

$BC = -158.78 \cos 40°0'$

$= -121.63$ (southing)

$CD = -117.67 \cos 45°0'$

$= -83.20$ (Southing)

$DA = L\cos\theta$

Departure

$AB = +96.98 \sin 30°30'$

$= +49.22$ (easting)

$BC = +158.78 \sin 40°0'$

$= +102.06$ (easting)

$CD = -117.67 \sin 45°0'$

$= -83.20$ (westing)

$DA = L\sin\theta$

For a closed traverse, the algebric sum of latitude and departures must equal to zero.

So,

$+83.40 - 121.63 - 83.20 + L\cos\theta = 0$

or $L\cos\theta = 121.43$ (1)

and $+49.22 + 102.06 - 83.20 + L\sin\theta = 0$

$L\sin\theta = -68.08$ (2)

Since the latitude is positive and departure is negative, the line DA lies in the NW quadrant

$$\tan\theta = \frac{68.08}{121.43}$$

$$= 0.5606$$

$$\theta = 29^\circ 16' 38''$$

Bearing of DA = N $29^\circ 16' 38''$ W

$$\text{Length of DA} = \sqrt{(121.43)^2 + (68.08)^2}$$

$$= 139.21 \text{ m.}$$

Problem 07 :-

The following observations were taken from traverse stations A and B to points C and D by means of a stadia tacheometer fitted with an anallatic lens, the instrument constant being 100.

Inst. station	Staff station	Height of inst.	Bearing	Vertical angle	Staff reading
A	C	1.48	$126^\circ 30'$	$+12^\circ 10'$	0.77, 1.60, [illegible]
B	D	1.42	$184^\circ 45'$	$-10^\circ 30'$	0.56, 1.84, [illegible]

Coordinates of A = 112.8 N, 106.4 W

Coordinates of B = 198.5 N, 292.6 W

Determine the length of the line CD.

Solution :-

$$\text{Distance AC} = 100 \times (2.43 - 0.77) \times \cos^2 12^\circ 10'$$

$$= 158.68 \text{ m}$$

Distance BD = $100(2.82-0.86) \times \cos^2 10°30'$

$= 189.49$ m

Reduced bearing of AC = S 53°30' E

Reduced bearing of BD = S 4°45' W

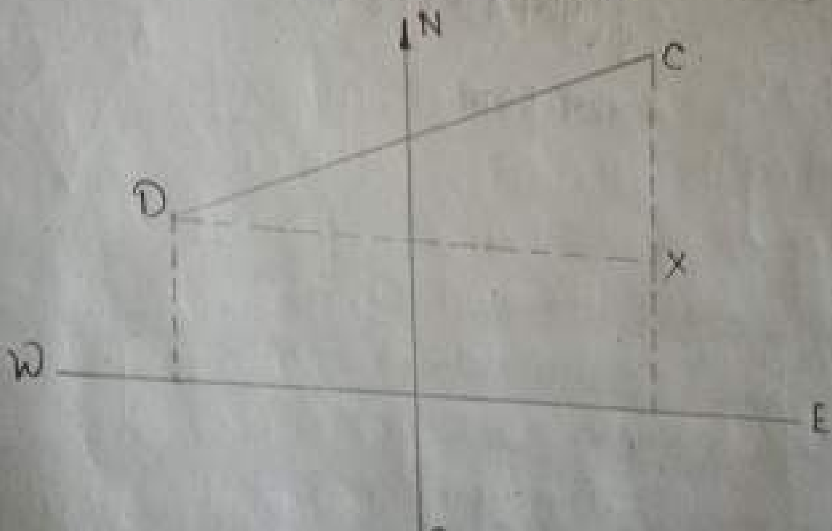
Line	Latitude	Departure
AC	$-158.63 \cos 53°30'$ $= -94.36$ m	$+158.63 \sin 53°30'$ $= +127.52$ m
BD	$-189.49 \cos 4°45'$ $= -188.84$ m	$+189.49 \sin 4°45'$ $= -15.69$ m

Coordinates of C

Latitude of A = + 112.82 m (northing)

Total latitude of C = $+112.83 - 94.36$

$= 18.44$ m



(fig 11.6)

Departure of A = -106.4 m (westing)

Total departure of C = $-106.40 + 127.52$

$= +21.12$ m

Coordinates of D

Latitude of B = +198.5 m (northing)

Total latitude of D = +198.50 − 188.84

= +9.66 m

Departure of B = −292.6 m (westing)

Total departure of D = −292.6 − 15.69

= 308.29 m

Dx = departure of C + departure of D

= 21.12 + 308.29

= 329.41 m

Cx = Latitude of C − latitude of D

= 18.44 − 9.66

= 8.78 m.

$$\text{Length } CD = \sqrt{(Dx)^2 + (Cx)^2}$$

$$= \sqrt{(329.4)^2 + (8.78)^2}$$

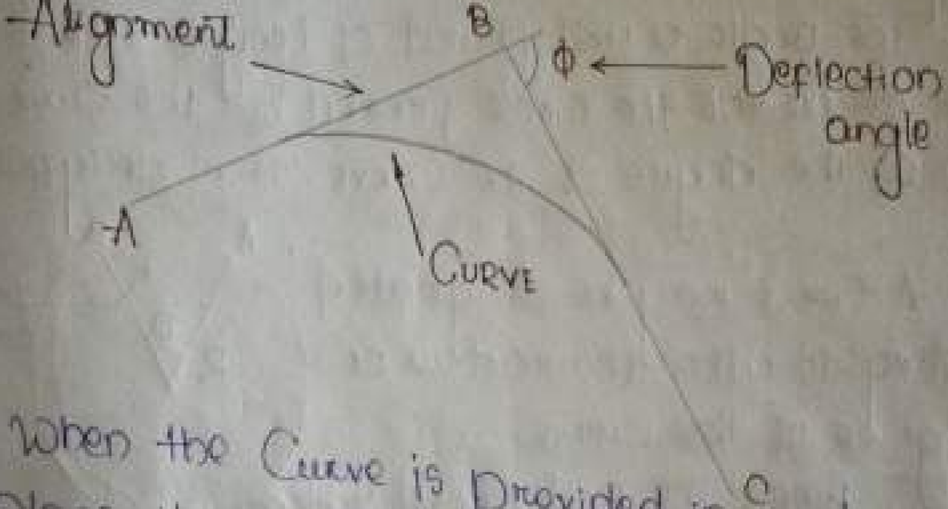
$$= 329.52 \text{ m.}$$

CHAPTER-03 Curves

Dt. 12.05.21

INTRODUCTION :-

During the survey of the alignment of a project involving roads or railways, the direction of the line may change due to some unavoidable circumstances. The angle of the ~~line~~ change in direction is known as the deflection angle for it to be possible for a vehicle to run easily along the road or railway track, the two straight lines (the original line & the deflected line) are connected by an arc which is known as the curve of the road or track.



When the curve is provided in the horizontal plane, it is known as a horizontal curve.

Again, along the alignment of any Project the nature of the ground may not be ~~followed by falling gradient and vice versa~~ uniform and may consist of different gradients (for instance, rising gradient may be followed by falling gradient and vice versa). In such a case, a parabolic curved path is provided in the vertical plane in order to connect the gradients for easy movement of the vehicles.

The Curve is known as a Vertical Curve. The following are the different forms of Curves:

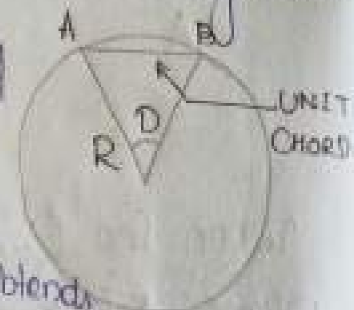
Curve
- Horizontal Curve
  - Simple Curve
  - Compound Curve
  - Reverse Curve
  - Transition Curve
  - Lemniscate Curve
- Vertical Curve

* **Definitions and Explanations of different terms:**

① **DEGREE OF CURVE :-**

The angle a unit chord of length 30 m subtends at the centre of the circle formed by the Curve is known as the degree of the Curve. It is designated as D.

-A Curve may be designated according to either the radius or the degree of the Curve.



When the unit Chord subtends an angle of 1°, it is called a one-degree curve, when the angle is 2°, a two-degree Curve, and so on.

It may be calculated that the radius of a one-degree Curve is 1,719 m.

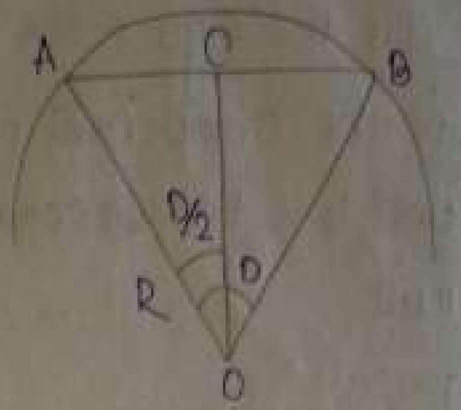
② **RELATION BETWEEN RADIUS AND DEGREE OF CURVE :-**

Let AB be the unit chord of 30 m, O the centre, R the radius and D the degree of the Curve.

Here, $OA = R$

$AB = 30\,m \quad AC = 15\,m$

$\angle AOC = \frac{D}{2}$



from triangle OAC,

$$\sin\frac{D}{2} = \frac{AC}{OA} = \frac{15}{R}$$

$$R = \frac{15}{\sin D/2}$$

When D is very small, sin D/2 may be taken as D/2 radians.

$$\therefore R = \frac{15}{(D/2)\times(\pi/180)} = \frac{15\times 360}{\pi D} = \frac{1{,}718.9}{D} = \frac{1{,}719}{D}$$

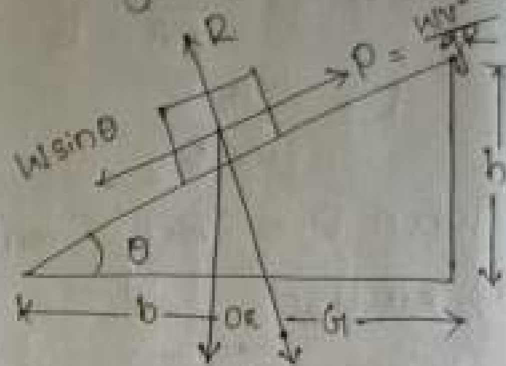
(approx)

(3) Superelevations

When a particle moves in a circular path, then a force (known as centrifugal force) acts upon it, and tends to push it away from the centre. Similarly, when a vehicle suddenly moves from straight to a curved path, the centrifugal force tends to push the vehicle away from the road or track. This is because there is no component force to counterbalance this centrifugal force. To counterbalance the centrifugal force, the outer edge on the road or rail is raised to some height (w.r.t inner edge), so that the sine component of the weight of the vehicle ($W\sin\theta$) may counterbalance the overturning force. The height through which the outer edge of the road or rail is raised is known as superelevation or Cant.

In the figure the p is the centrifugal force, W is the component of the weight of the vehicle, and h is the superelevation given to the road or rail.

For equilibrium,



$$W \sin\theta = \frac{Wv^2}{gR}$$

or $$W \times \frac{h}{b} = \frac{Wv^2}{gR}$$ (when θ is very small, $\sin\theta = \tan\theta = h/b$)

or

$$h = \frac{bv^2}{gR} \text{ for roads (1)}$$

or

$$h = \frac{Gv^2}{gR} \text{ for railways (2)}$$

Where,

b = width of the road in metres

G = distance between centres of rails (gauge) in metres

R = radius of the curve in metres

g = acceleration due to gravity = $9.8\ m/s^2$

v = speed of the vehicle in metres per sec

h = superelevation in metres.

(4) Centrifugal ratio

This ratio is between the centrifugal force and the weight of the vehicle is known as centrifugal ratio.

centrifugal ratio (CR) $= \frac{P}{W} = \frac{Wv^2}{gR \times W} = \frac{v^2}{gR}$

Allowable value for CR in roads $= \frac{1}{4}$

Allowable value for CR in railways $= \frac{1}{8}$

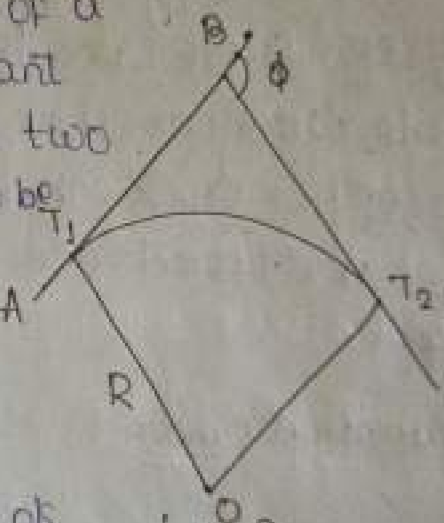
TYPES OF HORIZONTAL CURVES :-

D1-May 15-2021

The following are the different types of horizontal Curves :-

① **Simple Circular Curve :**

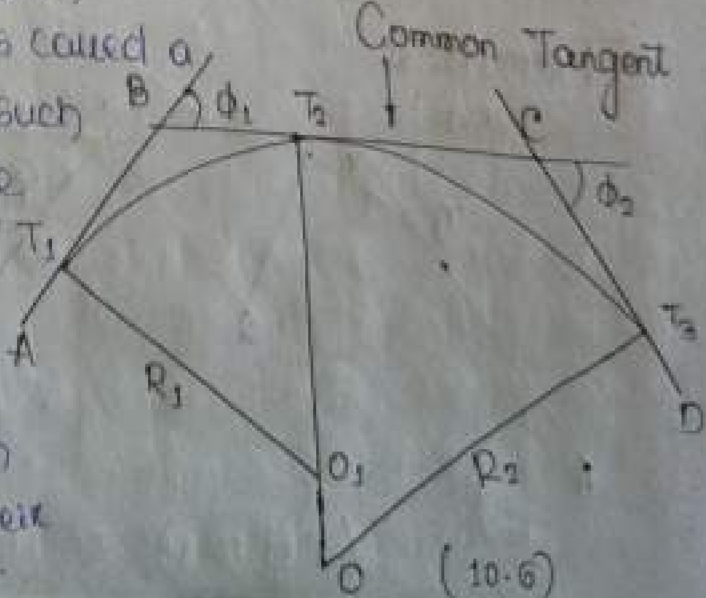
When a Curve Consists of a single arc with a constant radius connecting the two tangents, it is said to be a circular curve.



(10.5)

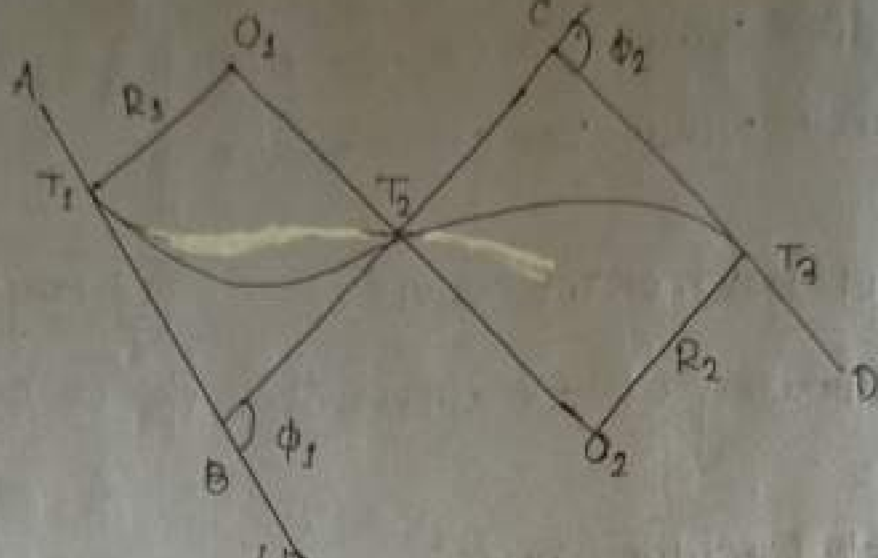
② **Compound Curve :**

When a curve consists of two or more arcs with different radii, it is called a Compound curve. Such a curve lies on the same side of a common tangent & the centres of the different arcs lie on the same side of their respective tangents.



(10.6)

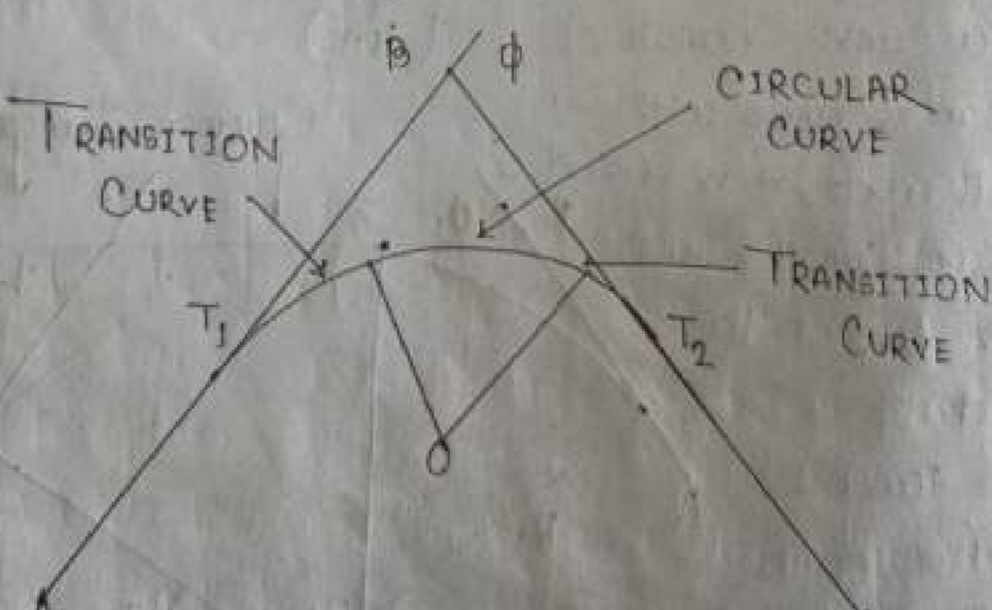
③ Reverse curve



(Fig 10.7)

A reverse curve consists of two arc bending opposite directions. Their centres lie on opposite sides of the curve. Their radii may be either equal or different, and they have one common tangent.

④ Transition Curve

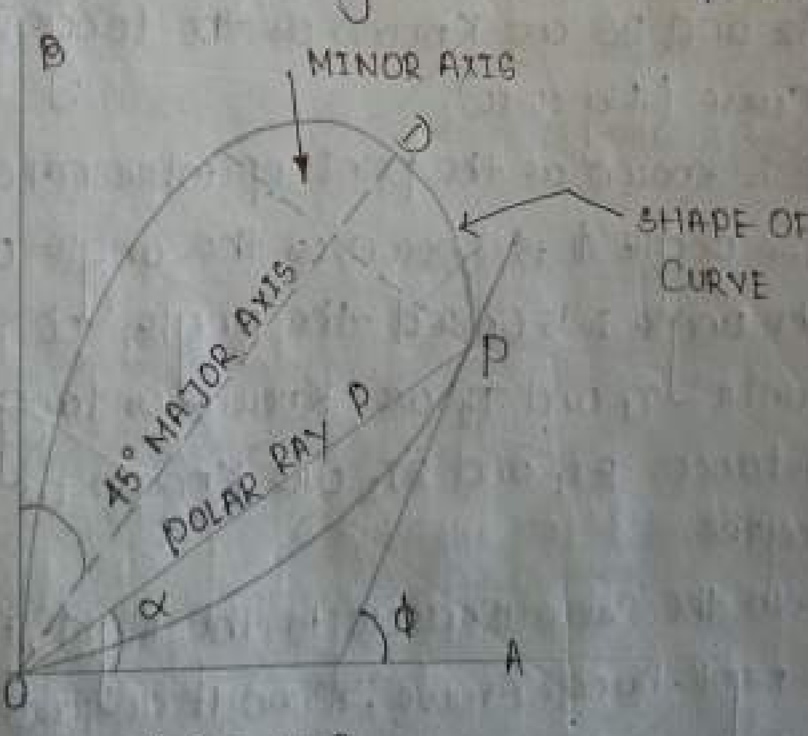


(fig 10.8)

A curve of variable radius is known as a transition curve. It is also called a spiral curve or easement curve. In railways, such a curve is provided on both sides of a circular curve to minimise superelevation. Excessive superelevation may cause wear and tear of the rail section and discomfort to passengers.

(5) Lemniscate curve

A lemniscate curve is similar to a transition curve, and is generally adopted in city roads were the deflection angle is large. In this fig(10.9) OPD shows the shape of such a curve. The curve is designated by taking a major axis OD, minor axis PP', with origin O and axes OA and OB. OP(p) is known as the polar ray and α as the polar angle.



(fig 10.9)

Considering the properties of polar Coordinates the polar equation of the Curve is given by where,

$$r = \frac{p}{3 \sin 2a}$$

p = polar ray of any point

r = radius of curvature at that point

a = polar deflection angle

At the origin, the radius of curvature is infinite. It then gradually decreases and becomes minimum at the apex D.

Length of Curve OPD = $1.3115 K$

$K = 3r\sqrt{\sin 2a}$

NOTATION USED WITH CIRCULAR CURVES :-

(1) AB and BC are known as the tangents to the curve (fig 10.10).

(2) B is known as the point of intersection or vertex

(3) The angle ϕ is known as the angle of deflection

(4) The angle I is called the angle of intersection

(5) Points T_1 and T_2 are known as tangent points

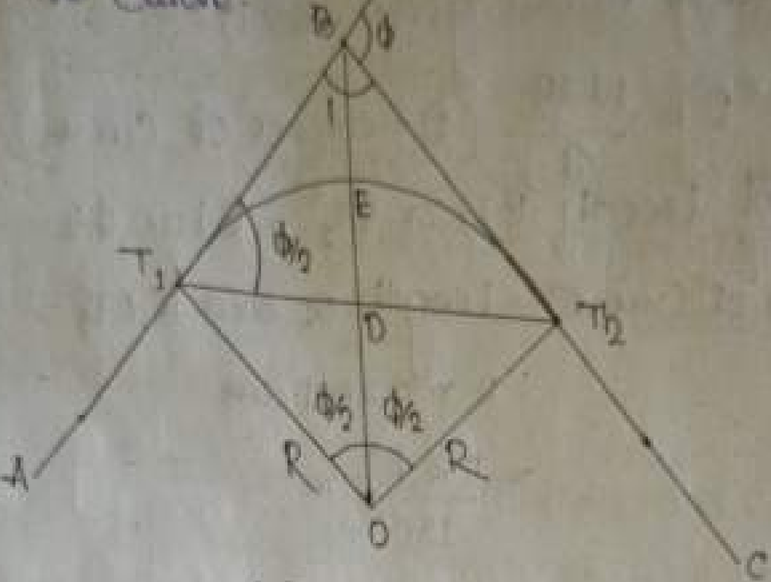
(6) Distances BT_1 and BT_2 are known as tangent lengths.

(7) When the curve deflects to the right, it is called a right-hand curve, when it deflects to the left, it is said to be a left-hand curve.

(8) AB is called the rear tangent and BC is the forward tangent.

(9) The straight line T_1DT_2 is known as the long chord.

(10) The curved line T_1ET_2 is said to be the length of the curve.



(Fig 10.10)

(11) The mid-point E of the curve T_1ET_2 is known as the apex or summit of the curve.

(12) The distance BE is known as the apex distance or external distance.

(13) The distance DE is called the versed sine of the curve.

(14) R is the radius of the curve.

(15) $\angle T_1OT_2$ is equal to the deflection angle ϕ.

(16) The point T_1 is known as the beginning of the curve or the point of curve.

(17) The end of the curve (T_2) is known as the point of tangency.

PROPERTIES OF SIMPLE CIRCULAR CURVE :-

Consider fig. 10.10

① If the angle of intersection is given, then

$\phi = 180^\circ - I$ (I = angle of intersection)

② If radius is not given, then

$R = \frac{1,719}{D}$ (D = degree of curve)

③ Tangent length BT_1 or $BT_2 = R \tan \phi/2$

④ Length of curve = Length of arc T_1ET_2

$= R \times \phi$ radians

$= \frac{\pi R \phi^\circ}{180^\circ}$ m

Again,

length of curve $= \frac{30\phi}{D}$ (if degree of curve D is given)

⑤ Length of long chord $= 2T_1D = 2OT_1 \sin \phi/2$

$= 2R \sin \phi/2$ m

⑥ Apex distance $= BE = OB - OE$

$= R \sec \phi/2 - R$

$= R(\sec \phi/2 - 1)$ m

⑦ Versed sine of curve $= DE = OE - OD$

$= R - R\cos \phi/2$

$= R(1 - \cos \phi/2)$ m

⑧ Full chord (peg interval): pegs are fixed at regular intervals along the curve. Each interval is said to equal the length of a full chord or unit chord. The curve is represented by a series of chords, instead of arcs. Thus, the length of the chord is partically equal to the length of the arc. In usual practice, the length of the unit chord should not be more than $1/20$th of the radius of the curve.

In railways curves, the unit chords (peg intervals) are generally taken between 20 and 30 m. In road curves, the unit chords should be 10 m or less. It should be remembered that the curve will be more accurate if short units chords are taken.

(9) Initial subchord: sometimes the chainage of the first tangent point works out to be a very odd number. To make it a round number, a short chord is introduced at the beginning. This short chord is known as the initial subchord.

(10) final Subchord: sometimes it is found that after introducing a number of full chords, some distance still remains to be covered in order to reach the second tangent point. The short chord is introduced for covering this distance is known as the final subchord.

(11) Chainage of first tangent point

= Chainage of intersection point − tangent length

(12) Chainage of second tangent point

= chainage of first tangent point + Curve length

HORIZONTAL CURVE SETTING BY CHAIN-AND-TAPE METHOD :-

The following are the general methods employed for setting out curves by chain and tape :

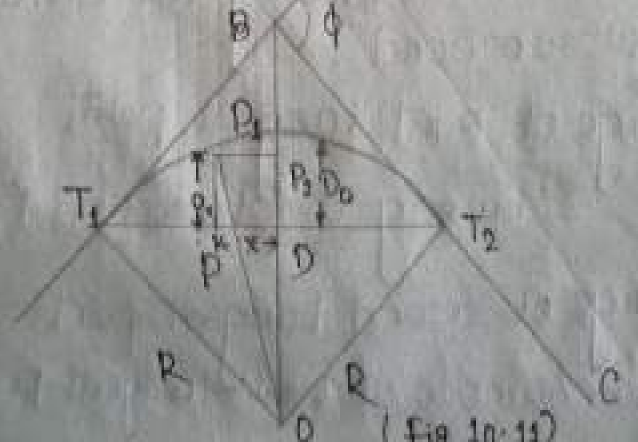
(1) Taking offsets or ordinates from the long chord.
(2) Taking offsets from the chord produced.
(3) Successively bisecting the arcs.
(4) Taking offsets from the tangents.

A. Offsets or ordinates from Long Chord :-

Let AB and BC be two tangents meeting at a point B, with a deflection angle ϕ. The following data are calculated for setting out the curve (Fig 10.11).

(1) The tangent length is calculated according to the formula ; $TL = R \tan \phi/2$
(2) Tangent points T_1 and T_2 are marked.
(3) The length of the curve is calculated according to the formula.

$$CL = \frac{\pi R \phi^{\circ}}{180^{\circ}}$$



(Fig 10.11)

(4) The chainages of T_1 and T_2 are found out.

(5) The length of the long chord (L) is calculated from:

$$L = 2R \sin \phi/2$$

(6) The long chord is divided into two equal halves (the left half and the right half). Here the curve is symmetrical in both the halves.

(7) The mid-ordinate O_0 is calculated as follows:

(a) $O_0 = DE$ = versed sine of curve $= R(1 - \cos \phi/2)$ ... (1)

(b) Again $OF = R$ and $OD = R - O_0$

From triangle OT_1D, $OT_1^2 = OD^2 + T_1D^2$

or

$$R^2 = (R - O_0)^2 + \left(\frac{L}{2}\right)^2$$

or

$$R - O_0 = \sqrt{R^2 - (L/2)^2}$$

or

$$O_0 = R - \sqrt{R^2 - (L/2)^2} \quad \text{.......} (2)$$

Thus, the mid-ordinate O_0 can be calculated from Eq. (1) or (2).

(8) Considering the left half of the long chord, the ordinates O_1, O_2 .... are calculated at the distances x_1, x_2 ..... taken from D towards the tangent point T_1.

The formula for the calculation of ordinates is deduced as follows;

Let P be a point at a distance x from D. Then PP_1 (O_x) is the required ordinate.

A line P_1P_2 is drawn parallel to T_1T_2 from the triangle OP_1P_2.

$$OP_1^2 = OP_2^2 + P_1P_2^2$$

or

$$R^2 = \{(R-O_0+O_x)\}^2 + x^2 \quad [\text{where, } OP_2 = (R-O_0)+O_x]$$

or

$$R-O_0+O_x = \sqrt{R^2-x^2}$$

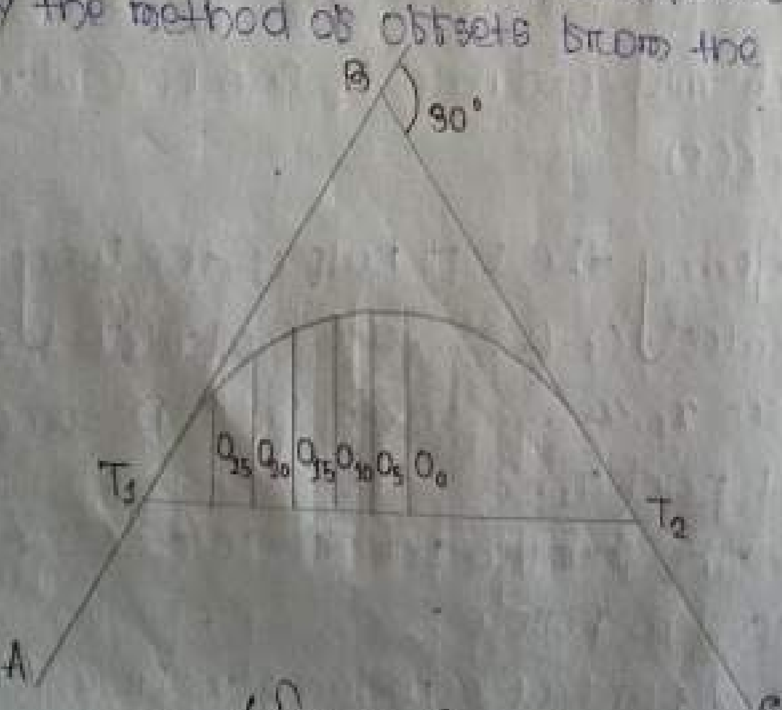
or

$$O_x = \sqrt{R^2-x^2} - (R-O_0)$$

(9) The ordinates for the right half are similar these obtained for the left half.

<u>Example :-</u>

Two tangents AB and BC ~~interest~~ intersect at a point B at chainage 150.5 m. Calculated all the necessary data for setting out a circular curve of radius 100m and deflection angle 30° by the method of offsets from the long chord.



(fig 10.1)

Solution-

(1) Tangent length = $R \tan\frac{\phi}{2} = 100 \times \tan 15° = 26.79$ m.

(2) Chainage of $T_1 = 150.50 - 26.79 = 123.71$ m

(3) Curve length $= \frac{\pi R \phi°}{180°}$

$= \frac{3.14 \times 100 \times 30°}{180°} = 52.36$ m.

(4) Chainage of $T_2 = 123.71 + 52.36$

$= 176.07$ m.

(5) Length of long chord $(L) = 2R \sin\phi/2$

$= 2 \times 100 \times \sin 15°$

$= 51.76$ m

(6) The long chord is divided into two equal halves

Each half $= \frac{1}{2} \times 51.76 = 25.88$ m.

(7) Mid-ordinate, $O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

$= 100 - \sqrt{100^2 - 25.88^2}$

$= 3.41$ m.

(8) The ordinates are calculated at 5 m intervals starting from the centre towards T_1 for the left half.

$O_5 = \sqrt{R^2 - x^2} (R - O_0)$

$= \sqrt{(100^2 - 5^2)} - (100 - 3.41)$

$= 99.87 - 96.59$

$= 3.28$ m

$O_{10} = \sqrt{100^2 - 10^2} - 96.59$

$= 99.50 - 96.59$

$= 2.91$ m.

$$O_{15} = \sqrt{((100)^2 - (15)^2)} - 96.59$$
$$= 98.87 - 96.59$$
$$= 2.28\ m.$$

$$O_{20} = \sqrt{(100^2 - 20^2)} - 96.59$$
$$= 97.97 - 96.59$$
$$= 1.38\ m.$$

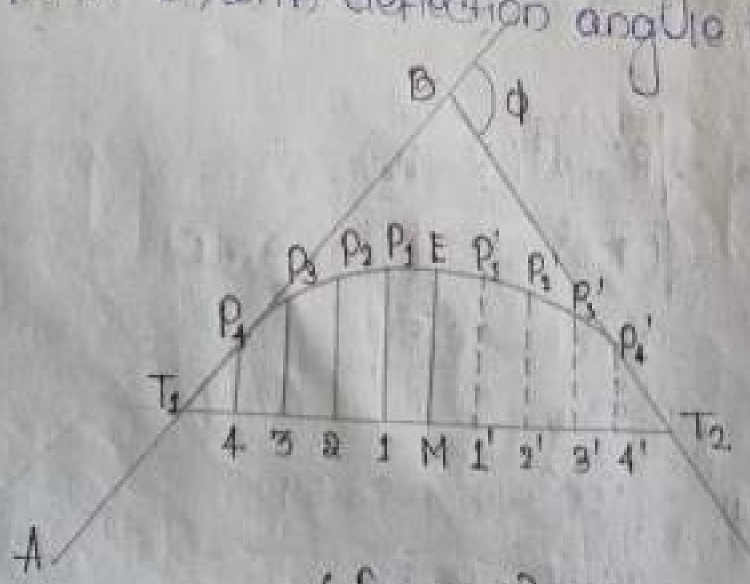
$$O_{25} = \sqrt{(100^2 - 25^2)} - 96.59$$
$$= 96.82 - 96.59$$
$$= 0.23\ m.$$

$$O_{25.88} = \sqrt{(100^2 - 25.88^2)} - 96.59$$
$$= 0 \text{ (checked)}$$

(9) The ordinates for the right half are similar to those for the left half.

Field Procedure for measuring ordinates by Long chord method :-

1) Let AB and BC be two tangents meeting at the point B, with deflection angle ϕ. (fig 10.12)



(fig 10.12)

(2) The tangent length is calculated from the usual formula, and points T_1 and T_2 are marked on the ground with pegs.

(3) The length of the long chord, T_1T_2 is calculated from the usual formula. The long chord is bisected at point M. The curve will be symmetrical on both sides of M.

(4) The ordinates are calculated for the left half at some regular intervals. Points 1, 2, 3 and 4 are marked with pegs along the long chord as shown in fig (10.12).

(5) Ordinates O_1, O_2, O_3 and O_4 are calculated from the usual formula.

(6) Perpendicular are set out at points 1, 2, 3 and 4. The calculated ordinates O_1, O_2, O_3 and O_4 identified along these perpendicular and points P_1, P_2, P_3 and P_4 are marked with pegs.

(7) In the right half, points 1', 2', 3' and 4' are marked with pegs and the corresponding ordinates (obtained for the left half) are set out to mark the points P_1', P_2', P_3' and P_4'.

(8) All these points P_1, P_2, ... and P_1', P_2' ... are on the curve. These points are joined by rope or thread to show the shape of the curve along the alignment (centreline) of the project.

B. Offsets from chord produced :

(1) Let AB = rear tangent T_1 = first tangent po...

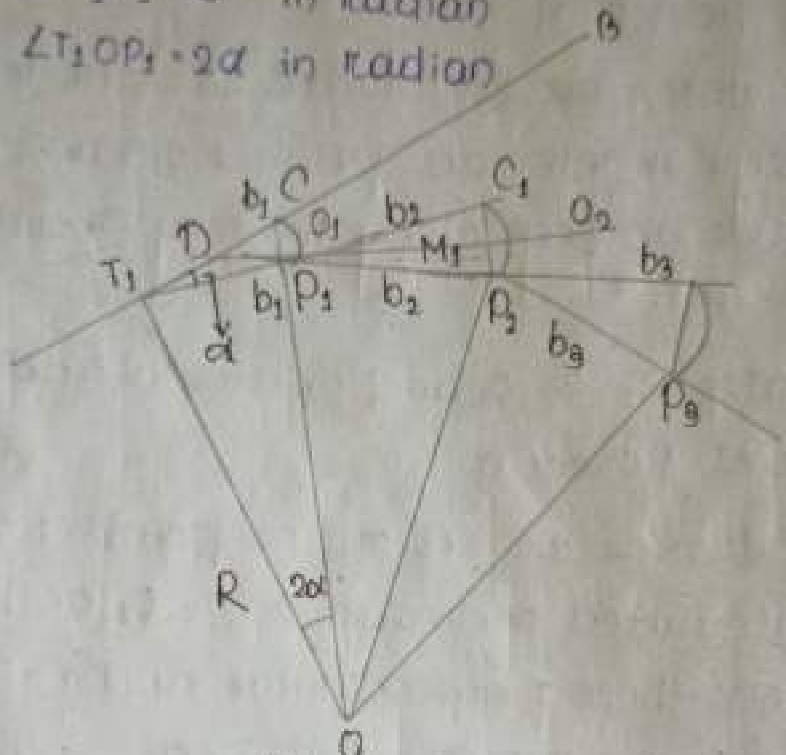
P_1 = first point on curve

$T_1C = T_1P_1 = b_1$, first chord or initial sub-ch...

$CP_1 = O_1$ = first offset

$\angle CT_1P_1 = \alpha$ in radian

$\therefore$ $\angle T_1OP_1 = 2\alpha$ in radian



(fig 10.13)

Now assuming that,

$$\text{chord } T_1P_1 = \text{arc } T_1P_1 = R \times 2\alpha$$

$$\alpha = \frac{T_1P_1}{2R}$$

Again,

$$\text{Chord } CP_1 \sim \text{arc } CP_1$$

so

$$\text{first offset } O_1 = CP_1 = T_1P_1 \times \alpha$$

$$\therefore \quad O_1 = \frac{T_1P_1^2}{2R} = \frac{b^2}{2R} \quad (T_1P_1 = b_1) \quad \ldots\ldots (1)$$

(2) Again, let P_2 be the next point on the Curve. The line T_1P_1 is extended and P_1C_1 is taken as the full chord b_2.

Now,

$$P_1C_1 = P_1P_2 = b_2$$

and Chord C_1P_2 = arc $C_1P_2 = O_2$ (second offset)

At P_1, a tangent is drawn which meets the rear tangent AB and D and chord C_1P_2 at M_1.

Here,

$$\angle C_1P_1M_1 = \angle DP_1T_1 \text{ (opposite)}$$

$$\angle DP_1T_1 = \angle DT_1P_1$$

So, $\angle C_1P_1M_1 = \angle DT_1P_1 = \angle DT_1P_1 = \angle CT_1P_1$

Triangles CT_1P_1 and $C_1P_1M_1$ are Similar

$$\therefore \frac{C_1M_1}{P_1C_1} = \frac{CP_1}{T_1P_1} \quad \text{or} \quad \frac{C_1M_1}{b_2} = \frac{O_1}{b_1}$$

or

$$C_1M_1 = \frac{b_2 \times O_1}{b_1} = \frac{b_2}{b_1} \times \frac{b_1^2}{2R} = \frac{b_1b_2}{2R}$$

Here,

M_1P_2 is the offset from the tangent at P_1.

So, according to Eq.(1),

$$M_1P_2 = \frac{(P_1P_2)^2}{2R} = \frac{b_2^2}{2R}$$

Second offset, $O_2 = C_1P_2 = C_1M_1 + M_1P_2$

$$\therefore O_2 = \frac{b_1b_2}{2R} + \frac{b_2^2}{2R} = \frac{b_2(b_1+b_2)}{2R} \qquad (2)$$

(3) Similarly,

Third offset,

$$O_3 = \frac{b_3(b_2 + b_3)}{2R} = \frac{b_3^2}{R} \quad (\text{as } b_2 = b_3 = b_4 \ldots$$

fourth offset,

$$O_4 = \frac{b_4(b_3 + b_4)}{2R} = \frac{b_4^2}{R} \text{ and so on}$$

(4) Last offset,

$$O_n = \frac{b_n(b_{n-1} + b_n)}{2R}$$

where,

b_n = final sub-chord b_{n-1} = Last full chord

<u>Example :-</u>

Two tangents intersect at a chainage of 1,000m the deflection angle being 30°. Calculate all the necessary data for setting out a circular curve of radius 200m by the method of offsets from the chord produced, taking a peg interval of 20m.

<u>Solution</u>

<u>Given data :-</u>

$\phi = 30°$, $R = 200$m, chainage of intersection point = 1000m, and full chord = 20m.

(1) Tangent length $= R \tan \frac{\phi}{2}$

$= 200 \times \tan 15°$

$= 53.58$ m.

(2) Curve length $= \frac{\pi R \phi°}{180°} = \frac{\pi \times 200 \times 30}{180} = 104.72$m

(3) Chainage of first tangent point $= 1,000 - 53.58$

$= 946.42$ m.

(4) chainage of second tangent point = 946.42 + 104.72
= 1,051.14 m.

(5) Initial sub-chord = 950.00 − 946.42 = 3.58 m.

(6) No. of full chords of length 20 m = 5

Chainage covered = 950.00 + 100.00
= 1,050.00 m.

(7) final sub-chord = 1,051.14 − 1,050.00
= 1.14 m.

(8) first offset for initial sub-chord,

$$O_1 = \frac{b_1^2}{2R}$$

$$\therefore O_1 = \frac{(3.58)^2}{2 \times 200} = 0.03 \text{ m.}$$

second offset for full chord,

$$O_2 = \frac{b_2(b_1+b_2)}{2R} = \frac{20(3.58+20)}{2 \times 200} = 1.18 \text{ m}$$

Third offset for full chord, $O_3 = \frac{b_3^2}{R} = \frac{20^2}{200} = 2.0 \text{ m}$

fourth offset for full chord, $O_4 = \frac{b_4^2}{R} = \frac{20^2}{200} = 2.0 \text{ m.}$

fifth offset for full chord, $O_5 = \frac{b_5^2}{R} = \frac{20^2}{200} = 2.0 \text{ m}$

sixth offset for full chord, $O_6 = \frac{b_6^2}{R} = \frac{20^2}{200} = 2.0 \text{ m.}$

Seventh offset for final sub-chord,

$$O_7 = \frac{1.14(20+1.14)}{2 \times 200} = 0.06 \text{ m.}$$

Note: There will be a total of seven offsets one for the initial sub-chord, five for full chords, and one for the final sub-chord. Here, the third through sixth offsets will be of the same length.

Field procedure for Setting out Curve by method of offsets from Chord produced:-

(1) Suppose AB and BC are the tangents, and B is the point of intersection. (fig 10.14)

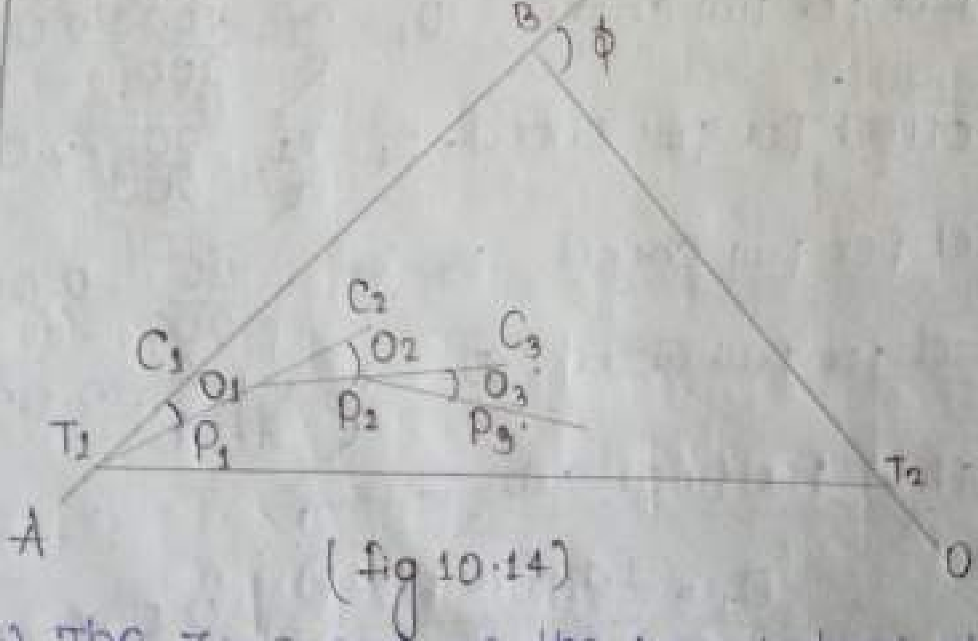
(2) By calculating the tangent length, points T_1 and T_2 are marked on the ground with pegs.

(3) The Curve length is calculated, and then the chainages of T_1 and T_2 are found out.

(4) The lengths of the initial and final sub-chords and the number of full chords are determined.

(5) The offsets for the initial sub-chord, full chord and final sub-chord are Calculated.

(6) The distance T_1C_1 is marked along the rear tangent AB so that T_1C_1 is equal to the initial sub chord.



(fig 10.14)

(7) The zero end of the tape is held at T_1, and an arc of radius T_1C_1 is drawn from this arc, a distance C_1P_1 is cut off as the first offset (O

(8) The line T_1P_1 is now extended by a distance P_1C_2, which is the second chord (i.e. a full chord).

(9) Then the zero end of the tape is held at P_1, and an arc of radius P_1C_2 is drawn. From this arc, a distance C_2P_2 is cut off as the second offset (O_2).

(10) This processes is continued until the second tangent point T_2 is reached.

(11) The last point should coinside with T_2. If it doesn't, the amount of error is found out. If the error is large, the entire operation should be repeated.

If the mirror is small, all the points are moved by sideways by an amount proportional to the square of their distances from T_1. The error is thus distributed among all the points of the curve.

(c) Successive Bisection of Arcs :-

(1) In fig. 10.15 AB and BC are two tangents intersecting at B, the deflection angle being ϕ. The tangent length is calculated, and tangent points T_1 and T_2 are marked on the ground with pegs.

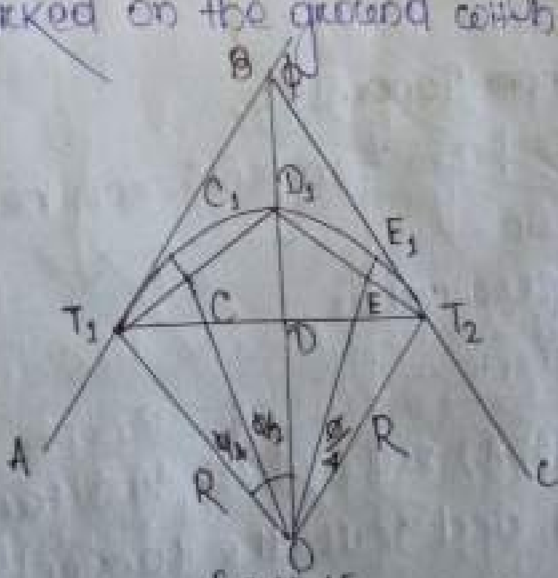


fig 10.15

(2) T_1T_2 is the length of the long chord, which is bisected at D. A perpendicular is set out at this point and a distance DD_1 is cut off which is equal to the versed sine of the curve.

$$DD_1 = \text{Versed sine of Curve} = R(1 - \cos \phi/2)$$

(3) Again the lengths T_1D_1 and T_2D_1 will serve as long chords for the curves between T_1 and D_1.

(4) The distances T_1D_1 and T_2D_1 are measured and bisected at C and E. Now, the distances CC_1 & EE_1 will be equal to the versed sines of the small curve, which is given by

$$CC_1 = EE_1 = R(1 - \cos \phi/4)$$

The calculated distances CC_1 and EE_1 are cut off along the perpendicular drawn at C and E.

(5) This process is continued until the bisection of chords is not practically possible.
Then the points on the curve are joined by free hand.

D) Offsets from Tangents :-

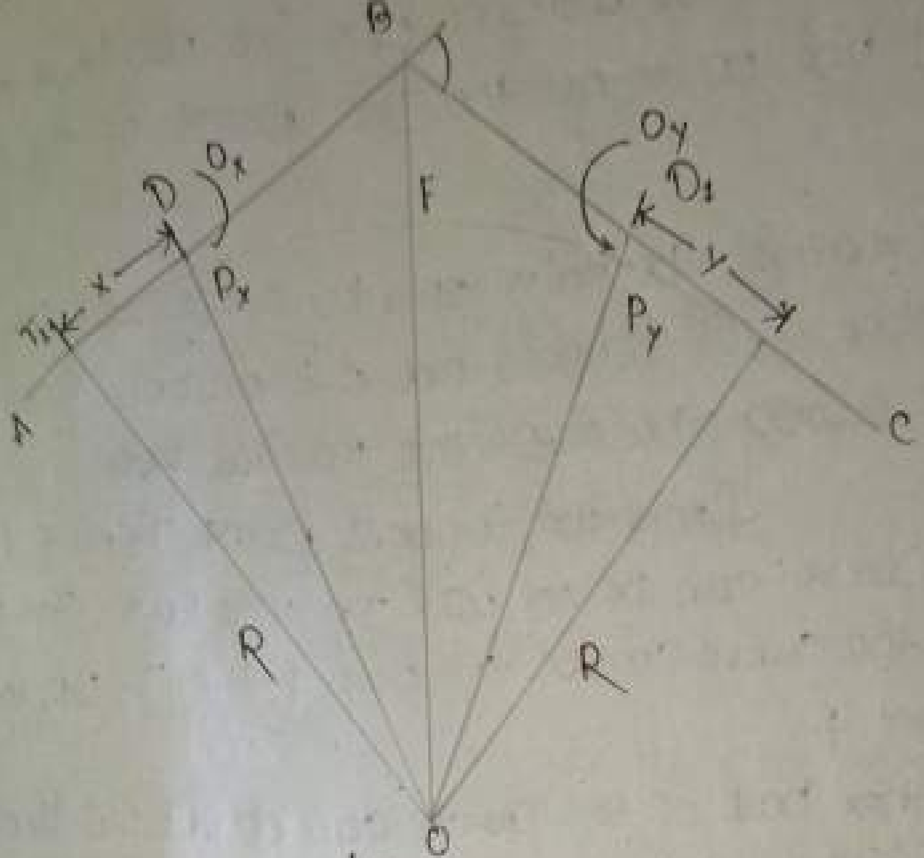
Offsets from tangents may be :

1. Radial
2. Perpendicular

(1) <u>Radial offsets</u> -

(a) In fig. 10.16, AB and BC are two tangents intersecting at B, and that the tangent points are

T_1 and T_2.



(Fig 10.16)

Let us take a point D on the rear tangent AB such that

$$T_1D = x.$$

Let O_x be the radial offset at D.
The point D is joined with the centre O. So, OD is the radial line.
Now, from ΔT_1OD

$$OT_1^2 + T_1D^2 = OD^2$$

Where,

$$OT_1 = R$$
$$OD = R + O_x$$
$$T_1D = x$$

$$\therefore R^2 + x^2 = (R + O_x)^2$$

or $\quad R + O_x = \sqrt{R^2 + x^2}$

$$O_x = \sqrt{R^2 + x^2} - R$$

(b) The calculated distance O_x is cut off from the radial line OD to get the first point of the curve P_x.

(c) By increasing the value of x by a regular amount a number of offsets are obtained. these are set off along the respective radial lines.

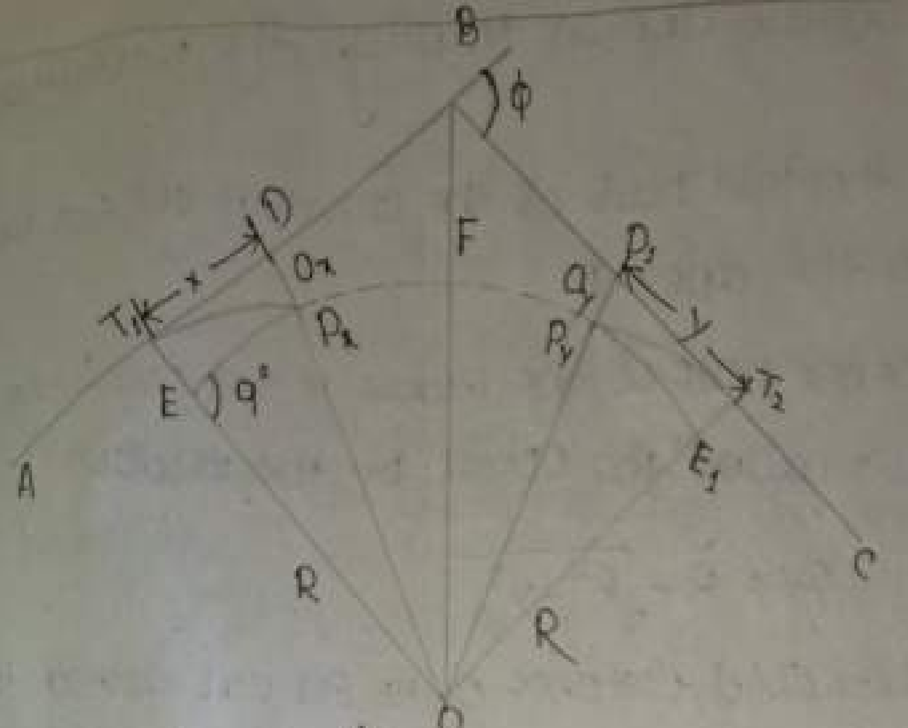
From the tangent point T_1, one half of the curve can be set out. In this case, the left half of the curve can be set out from T_1 up to the apex point F.

(d) The other half of the curve can be set out from the second tangent point T_2. Let a point D_1 be taken at a distance y from T_2. the offset O_y is then calculated as $O_y = \sqrt{R^2 + y^2} - R$

The calculated distance O_y is set off along the radial line OD_1 to get the point P_y on the curve. Thus by increasing the value of y, the required offsets are calculated and set off along their respective radial lines to get the points on curve for the right half.

(2) <u>By Perpendicular Offsets :</u>

(a) In fig. 10.17 AB and BC are two tangents meet at a point B. The tangent length is calculated and the tangent points T_1 and T_2 are marked on the ground.



(fig 10.17)

(b) A point D is taken along the rear tangent AB at a distance x from T_1. Let O_x be the perpendicular offset at D. The line EP_x is drawn parallel to T_1D.

Here $OE = R - O_x$, $OP_x = R$, $EP_x = x$

from $\triangle OEP_x$, $OP_x^2 = EP_x^2 + OE^2$

or $R^2 = x^2 + (R - O_x)^2$

or $R - O_x = \sqrt{R^2 - x^2}$

or $O_x = R - \sqrt{R^2 - x^2}$

(c) This calculated distance O_x is set out along the perpendicular drawn at L, to get the point P_x on the curve.

Similarly by progressively increasing the value of x by a regular amount a series of offsets are obtained. These are set out along the

Perpendicular drawn through the respective points.

Thus the left half of the curve is set out from T_1 up to the apex F.

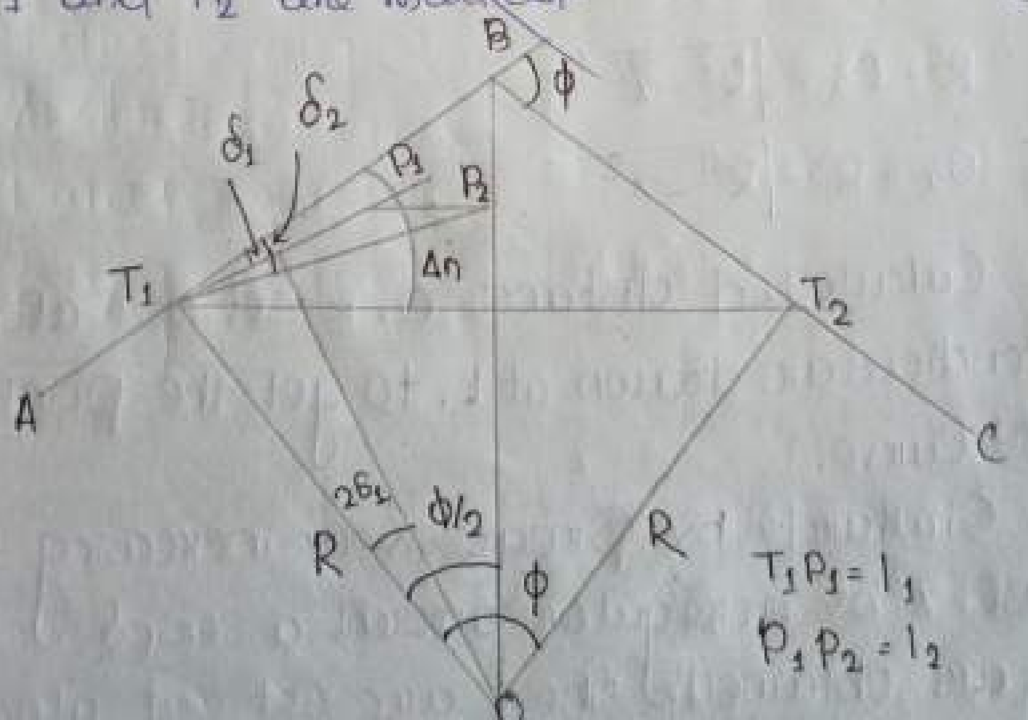
(d) The other half of the curve is set out from T_2, by calculating the offset by the relation

$$O_y = R - \sqrt{R^2 - y^2}$$

The calculated distance O_y is set out along the perpendicular drawn at D_1 to get the point P_y on the curve. This process is continued until the apex F is reached.

INSTRUMENTAL METHOD – HORIZONTAL CURVE SETTING BY DEFLECTION ANGLE METHOD OR RANKINE'S METHOD

Let AB and BC two tangents intersecting at B, the deflection angle being ϕ (fig 10.18). The tangent length is calculated and tangent points T_1 and T_2 are marked.



$T_1P_1 = l_1$

$P_1P_2 = l_2$

Let,

P_1 = first point on the Curve,

$T_1P_1 = l_1$ Length of first chord (initial Sub-chord)

δ_1 = deflection angle for first chord

R = radius of the Curve

Δ_n = total deflection for the chords

Here,

$$\angle T_1OP_1 = 2 \times \angle BT_1P_1 = 2\delta_1$$

Again,

$$\text{Chord } T_1P_1 = \text{arc } T_1P_1$$

Now,

$$\frac{\angle T_1OP_1}{l_1} = \frac{360^\circ}{2\pi R}$$

or

$$2\delta_1 = \frac{360^\circ \times l_1}{2\pi R}$$

or

$$\delta_1 = \frac{360^\circ \times l_1}{2 \times 2\pi R} \text{ degrees}$$

or

$$= \frac{360 \times 60 \times l_1}{2 \times 2 \times \pi R} \text{ mins}$$

$$= \frac{1,718.9 \times l_1}{R} \text{ mins}$$

Similarly,

$$\delta_2 = \frac{1,718.9 \times l_2}{R} \text{ mins}$$

$$\delta_3 = \frac{1,718.9 \times l_3}{R} \text{ mins} \quad \text{and so on.}$$

finally,

$$\delta_n = \frac{1,718.9 \times l_n}{R} \text{ mins}$$

Again, when degree of curve D is given,

$$\delta_1 = \frac{D \times l_1}{60} \text{ degrees}$$

$$\delta_2 = \frac{D \times l_2}{60} \text{ degrees and so on.}$$

finally,

$$\delta_n = \frac{D \times l_n}{60} \text{ degrees}$$

Arithmetical check: $\delta_1 + \delta_2 + \delta_3 + \cdots + \delta_n = \Delta_n = \phi/2$

Steps to remember for Calculating data

(1) Tangent length
(2) Curve length
(3) Chainage of first tangent point
(4) Chainage of second tangent point
(5) Initial sub-chord
(6) Number of full chords
(7) final sub-chord
(8) Deflection angle for initial sub-chord
(9) Deflection angle for full chord
(10) Deflection angle for final sub-chord
(11) Arithmetical check
(12) Data for field check
(13) setting out table

Example-

Two tangents intersect at chainage 1,250m. The angle of intersection is 150°. Calculate all data necessary for setting out a curve of radius 250m by the deflection angle method. The peg intervals may be taken as 20m. Prepare a setting out table when the least count of the vernier is 20". Calculate the data for field checking.

solution

Given data :-

Radius = 250 m

Deflection angle $\phi = 180° - 150°$
$= 30°$

Chainage of intersection Point = 1,250m

Peg interval = 20 m

LC of vernier = 20"

① Tangent length = $R \tan \phi/2$
$= 250 \times \tan 15°$
$= 67.0\,m$

② Curve Length = $\frac{\pi R \phi°}{180°} = \frac{\pi \times 250 \times 30°}{180°} = 130.89\,m$

③ Chainage of first TP, $T_1 = 1{,}250.0 - 67.0 = 1{,}183.0\,m$

④ Chainage of second TP, $T_2 = 1{,}183.0 + 130.89 = 1{,}313.89\,m$

⑤ Length of initial sub-chord = $1{,}190.0 - 1{,}183.0 = 7.0\,m$

⑥ No. of full chords (20m) = 6

Chainage covered $= 1{,}190.0 + (6 \times 20)$
$= 1{,}310.00\,m$

⑦ Length of final sub-chord $= 1{,}313.89 - 1{,}310.00 = 3.89\,m$

(8) Deflection angle for initial sub-chord,

$$\delta_1 = \frac{1,718.9 \times 7.0}{250} \text{ mins}$$

$$= 0^\circ 48' 8''$$

(9) Deflection angle for full chord,

$$\delta = \frac{1,718.9 \times 20}{250} \text{ mins}$$

$$= 2^\circ 17' 31''$$

(10) Deflection angle for final sub-chord,

$$\delta_n = \frac{1,718.9 \times 3.89}{250} = 0^\circ 26' 45''$$

(11) ~~Arithmetical check:~~

Total deflection angle $(\Delta_n) = \delta_1 + 6 \times \delta + \delta_n$

$$\phi/2 = \frac{30^\circ}{2} = 15^\circ$$

~~Here,~~

$$\Delta_n = 0^\circ 48' 8'' + 6 \times 2^\circ 17' 31'' + 0^\circ 26' 45'' = 14^\circ 59' 59''$$

$$= 15^\circ \text{ (approximately)}$$

So, that calulated deflection angles are Correct

(12) Data for field check:

(a) Apex distance $= R(\sec \phi/2 - 1)$

$$= 250(\sec 15^\circ - 1)$$

$$= 8.82 \text{ m}$$

(b) Versed sine of Curve $= R(1 - \cos \phi/2)$

$$= 250(1 - \cos 15^\circ)$$

$$= 8.52 \text{ m}$$

Setting out Table:

Point						
T_1	1,183.0	—	—	—	—	starting point of Curve
P_1	1,190.0	7.0	0°48'8"	0°48'8"	0°48'0"	
P_2	1,210.0	20.0	2°17'31"	3°5'39"	3°5'40"	lc of vernier = 20"
P_3	1,230.0	20.0	2°17'31"	5°23'10"	5°23'0"	
P_4	1,250.0	20.0	2°17'31"	7°40'41"	7°40'40"	
P_5	1,270.0	20.0	2°17'31"	9°58'10"	9°58'0"	finishing Point on Curve
P_6	1,290.0	20.0	2°17'31"	12°15'43"	12°15'40"	
P_7	1,310.0	20.0	2°17'31"	14°33'14"	14°33'20"	
T_2	1,313.89	3.89	0°26'45"	14°59'59"	15°0'0"	

* * * * *

Basics of Scales & Basics of map

CHP-03
Dt 24.05.21

Scales :-

The ratio between the distances of two points on map, plan or photograph and the actual distance between the same two points on ground is called scale.

Representative fraction or Ratio scale or fractional scale :-

- The type of scale which shows the relationship between the map distances & the corresponding distance on ground in units of length.
- Rf is generally shown in fraction because it shows how much the real world is reduced to fit on the map.
- e.g:- fraction of 1:24,000 shows that one unit of length on map represents 24,000 of the same length on the ground i.e. mm, cm, inch.

Linear Scale

- This is also known as graphical scale or plain scale or bar scale.
- This is merely a straight line whose length is in certain proportion to the actual length on ground.
- It is divided into primary and secondary divisions.

Map

A selective, symbolised and generalised representation of the whole or part of the earth at a reduced scale is called map.

- Map is a symbolic representation of selected characteristic of a place usually drawn on a flat surface.
- Maps present information about the world in a simple visual way.
- They teach about world by showing size, shape of countries, location of features & distances between places.
- Maps can show distribution of things over earth such as settlement patterns etc.

Map scale

- All maps are scale models of reality. A maps scale indicates the relationship betn the distance on map and the actual distance on earth. This relationship can be expressed by a graphic scale, a vertical scale or a representative scale.
- On the basis of the scale maps are classified as

(a) Small scale map

These represents large areas eg: maps having scales of 1 cm = 400 km can be considered as small scale maps like wall maps or maps in school atlas.

(b) Large scale map

These maps represents small areas eg maps having scales of 1 cm = 2 m. Town plan cadastal maps showing boundaries of landed property etc are large scale map.

Map projection

→ It is a method of transferring the graticule of latitude & longitude on a plane surface.

→ It can also be defined as the transformation of spherical network of parallels and meridians on plane surface.

→ As we know earth is not flat but it is geoid in shape like sphere. The horizontal lines on globe represents parallel of latitude & the vertical lines represent ~~parallel~~ meridians is called graticule & drawing of graticule on a flat surface is called projection.

→ The need for map projection arises to have detailed study of a region which is not possible to do from a globe.

Dt - 31.05.2021

* How maps convey location & extent ?

→ A map is a collection of map elements laid out and organized on a page. Common map elements include the map frame with map layers, a scale bar, north arrow, title, descriptive text and a symbol legend.

→ The primary map element is the map frame and it provides the principal display of geographic information. Within the map frame geographical entities are presented as a series of map layers that a given map extent - for example, map layers as roads, rivers place names, buildings, political boundaries, surface elevation and satelite imagery

- The following graphic illustrates how geographical elements are portrayed in maps through a series of map layers. Map symbols and text are used to describe the individual geographic elements.
- Map layers are thematic representation of geographic information such as transportation, water and elevation.
- Map layers help convey information through:

* Describe features such as collection of points, lines and polygons.
* Map symbols, colours and labels that help to describe the objects in the map.
* Aerial photography or satellite imagery that covers the map extent.
* continuous surfaces such as elevation which can be represented in a number of ways - for example, as a collection of contour lines and elevation points or as shaded relief.

Spatial Relationship in a Map

- Maps help convey geographic relationship that can be interpreted and analyzed by map readers. Relationships that are based on location are reffered to as spatial relationships. Here are some examples :-

* Which geographic features connect to others (for example, water street connect with 15th Ave)
* Which geographic features are adjacent (contiguous) to others (for example, the city park is adjacent to

the university).

* Which geographic features are contained within an area (e.g. the building footprints are contained within the parcel boundary).
* Which geographic features are near others (Proximity) (for e.g. the Courthouse is near the state Capital)
* The feature geometry is equal to another feature (for eg. the city park is equal to the history site polygon).
* The difference in elevation of geographic features (for eg. the state Capital is uphill from the water)
* The feature is along another feature (for eg. the bus route follow along the street network).

- Within a map, such relationship are not explicity represented. Instead as the map reader you interpret relationship and derive information from the relative position and shape of the map elements such as the streets, contours, buildings, lakes and other features.
- In a GIS such relationships can be modeled by applying rich data types and behaviours and by applying a comprehensive set of spatial operators to the geographic objects.

Classification of maps

following are the broad classification maps.

(1) physical map

- A physical map is one that documents landscape features of a place. These maps generally show things like mountains, rivers, lakes.
- water bodies are generally shown in blue. The Mountains & elevation changes are sometimes shown with different colours & shades to show elevation. In physical map green usually indicates lower elevation while brown indicates higher elevation.

(2) Topographic maps

- A topographic map is similar to a physical map in that it shows different physical landscape features.
- Unlike physical map this map uses contour lines instead of colours to show changes in the landscape.
- Contour lines on map are normally spaced at regular intervals to show elevation changes.
- When contour lines are close together it means terrain is steep & vice-versa.

(3) Road maps

- It is one of the most widely used map types. These map shows major & minor highways & roads as well as things like airports, cities and points of interests such as parks & campgrounds & monuments.

→ Major highways in maps are generally shown in thick reds lines while minor roads are lighter in colour and narrower lines.

(4) Political Maps

→ A political map doesnot show topographic features like mountains. It focuses solely on the state & national boundaries of a place.

→ These maps also include the locations of cities large and small depending on the detail of the maps.

(5) Economic & resources maps

→ An economic & resources map shows the specific types of economic activity or natural resources present in an area through the use of different symbols or colours depending on what is being depicted.

(6) Thematic Maps

→ A thematic map is a map that focuses on a particular theme or special topic.

→ These maps are different from others as they don't just show rivers cities, political subdivisions, elevation & highways.

→ If these items appear on a thematic map, they are background informations and are used as the reference points to enhance the map theme.

(7) Climatic Maps

→ It shows information about the climate of an area.

- These maps can show things like the specific climate zones of an area based on temperature the amount of snow an area receives or the average no. of cloudy days.
- These maps generally use colours to show different climate areas.

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CHAPTER-04 Survey of India map Series June-07-

Survey of India (SOI) brings out two series of map through the national map policy, 2005

(1) Open Series maps (OSM):

→ OSM will be brought out exclusively by SOI. Primarily for supporting development activities in the country.

→ OSM bears different sheet number for maps and will be in UTM projection on WGS-84 datum.

→ Each of these OSMs (both in soft copy and hard copy, will become unrestricted after obtaining a one time clearance of the ministry of defence.

→ SOI will ensure that no civil & military vulnerable areas & points are shown on OSMs.

→ The SOI will issue from time to time detailed guidelines regarding all aspects of the OSM like procedure for access by user agencies, further dissemination/sharing of OSM amongst user agencies with or without value addition, ways & means of protecting business & commercial interests of SOI in data & other incidental matters.

→ User will be allowed to publish maps on hard copy and web with or without GIS data base. If the international boundary is depicted on the map, certification from SOI is necessary.

→ In addition, the SOI is currently preparing city maps. These city maps will be on large scale in WGS-84 datum & in public domain. The content of these maps will be decided by SOI in consultation with ministry of defence.

(2) Defence Series Maps (DSM) :-

- These will be the topographical maps (on everest / WGS-84 datum & Polyconic / UTM projection) on various scale.
- These will mainly cater for defence & national security requirements.
- This series of maps (in analogue or digital forms) for the entire country will be classified as appropriate and the guidelines regarding their use will be formulated by the ministry of defence.

INTRODUCTION TO AREAL PHOTOGRAPHY

We are familiar with photographs taken with normal cameras. These photographs provide us with a view of the object similar to the way we see them with our own eyes. In ~~order to~~ other word, we get a horizontal prospective of the object photographed. For example, a photograph of a part of settlement will provide us a perspective the way it appears to us when we look at it. Suppose we want to take a 'bird's eye view' of similar features, then we have to place ourselves somewhere in the air. When we do so and look down, we get a very different perspective. This perspective, which we get in aerial photographs, is termed as aerial photographs, is termed as aerial perspective.

The photographs taken from an aircraft or helicopter using a precision camera are termed aerial photographs.

- The photographs so obtained have been found to be indispensable tools in the topographical mapping and interpretation of the images of the objects

1. **Aerial Camera :-**
   A precision Camera specifically designed for use in aircrafts.

2. **Aerial film :-**
   A roll film with high sensitivity, high intrinsic resolution power and dimensionally stable emulsion support.

3. **Aerial Photography :-**
   Art, science and technology of taking aerial photographs from an air-borne platform.

(4) Aerial photograph:

A photograph taken from an air-borne platform using a precision camera.

(5) fiducial Marks:-

Index marks rigidly connected at the central or corner edges of the Camera body. when the film is exposed, these marks appears on the film negative.

(6) forward overlap:-

The common area on two successive photographs in the flight direction. It is usually expressed in Percen

(7) Image Interpretation:-

An act of identifying the images of the objects and judging their relative significance.

(8) Nadir point:-

The foot of the Perpendicular drawn from the Camera lens Centre on the ground plane

(9) Principal point:-

The foot of the perpendicular drawn from the Camera lens centre on the photo plane.

(10) Principal Distance:-

The perpendicular distance from the perspective Centre to the plane of the photograph.

(11) Perspective Centre:-

The point of the origin (Perspective Centre) of the bundle of light rays.

(12) photogrammetry:-

The science and technology of taking reliable measurements from aerial Photographs.

Uses of Aerial Photographs:

Aerial photographs are used in topographical mapping and interpretation. These two different uses have led to the development of photogrammetry and photo/image interpretation as two independent but related sciences.

photogrammetry

It refers to the science and technology of making reliable measurements from aerial photographs. The principles used in photogrammetry facilitate precise measurements related to the lengths, breadth and height from such photographs. Hence, they are used as the data source for creating and updating topographic maps.

Aerial photography in India:-

Aerial photography in India goes back to 1920 when large-scale aerial photographs of Agra city were obtained, subsequently, Air Survey party of the Survey of India took up aerial survey of Irrawaddy Delta forests, which was completed during 1923-1924. Subsequently, several similar surveys were carried out and advanced methods of mapping from aerial photographs were used. Today, aerial photography in India is carried out for the entire country under the overall supervision of the Directorate of Air Survey (Survey of India) New Delhi. Three flying agencies, i.e. Indian Air force, Air Survey Company, Kolkata and National Remote sensing Agency, Hydrabad have been officially authorised to take aerial photographs in India.

The procedure for intending aerial photographs for educational purposes could be made with ADFPS Party No. 73, Directorate of Air Survey, Survey of India, West Block IV, R.K. Puram, New Delhi.

Image Interpretation:

It is an art of identifying images of objects and judging their relative significance. The principles of image interpretation are applied to obtain qualitative information from the aerial photographs such as land use/land cover, topographical forms, soil types, etc. A trained interpreter can thus utilise aerial photographs to analyse the land-use changes.